Solved examples in degree-of-freedom analysis for single unit process

DEGREE-OF-FREEDOM ANALYSIS

Everyone who has done material balance calculations has had the frustrating experience of spending a long time deriving and attempting to solve equations for unknown process variables, only to discover that not enough information is available. Before you do any lengthy calculations, you can use a properly drawn and labeled flowchart to determine whether you have enough information to solve a given problem. The procedure for doing so is referred to as **degree-of-freedom analysis**.

To perform a degree-of-freedom analysis, draw and completely label a flowchart, count the unknown variables on the chart, then count the independent equations relating them, and subtract the second number from the first. The result is the number of degrees-of-freedom of the process. In general degree-of-freedom (DoF) analysis for steady state material balance problem is written as

Degree-of-Freedom = number of unknowns – numbers of independent equations

For material balance problems with chemical reactions, Degree-of-Freedom is written as

DoF = number of unknowns + number of independent reactions – number of independent material balance equations – auxillary relation (density relationship relating mass flow rate and volumetric flow rate, specified split – bottom and top product as well).

There are three possibilities:

If DoF = 0, the system is completely defined and you get a unique solution

If DoF > 0, the system is under defined and there are infinite number of solutions

If DoF < 0, the system is over defined and there are too many restrictions. Over defined problems cannot be solved to be consistent with all equations.

INDEPENDENT EQUATION

Equations are independent if you cannot derive one by adding and subtracting combinations of the others. For example, only two of the three equations x = 3, y = 2 and x + y = 5 are independent; anyone of them can be obtained from the other two by addition or subtraction.

In other words, a set of equations is independent if you cannot derive one by adding and subtracting combination of others.

(a) Is the following set of equation independent?

$$x + 2y + z = 1 \qquad (1)$$

$$2x + y - z = 2 \qquad (2)$$

$$y + 2z = 5 \qquad (3)$$

Solution

Yes, the above equations are independent because we cannot derive one by adding and subtracting the combination of others.

(b) Is the following set of equations independent?

$$x + 2y + z = 1 \qquad (1)$$

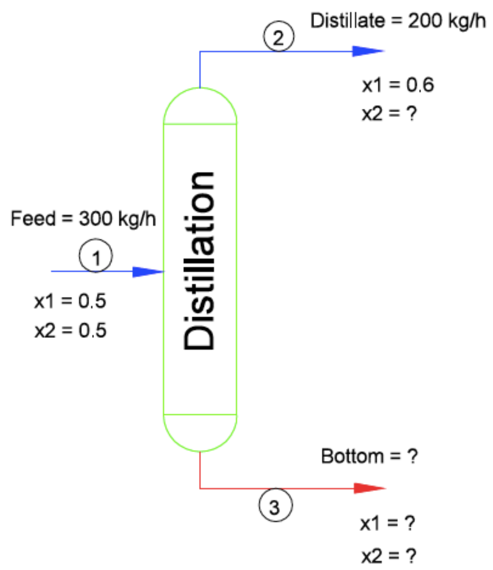
$$2x + y - z = 2 \qquad (2)$$

$$3x - 3y = 5 \qquad (3)$$

the above set of equations are not independent because we can derive (3) by adding equations (1) and (2)

Now we introduce the degree-of-freedom analysis for single unit operation where the steady state conditions prevail.

Problem 1: Perform a degree-of-freedom analysis for flow chart given below



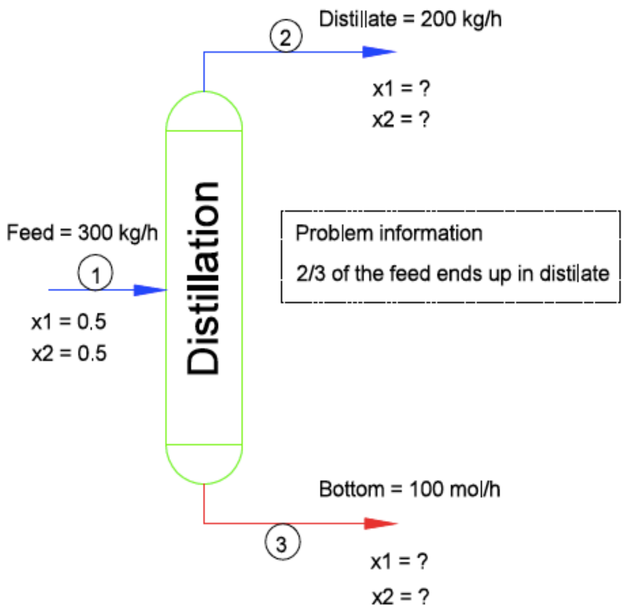
Number of unknowns (B, x_1,) = 2

Number of independent equations (two components 1,2) = 2

Number of auxillary relations = 0

Therefore, DoF = 2 -2 = 0

Problem 2: Perform a degree-of-freedom analysis for flow chart given below



Number of unknowns ($x_1$in Bottom and $x_1$in Distillate) = 2

Number of independent equations (two components 1,2) = 2

Number of auxillary relations (2/3 of the feed = 200 kg/s ends up in distillate which is already given in the flowchart may not be a useful relation.

Hence auxillary relation for this problem = 0

Therefore, DoF = 2 -2 = 0