

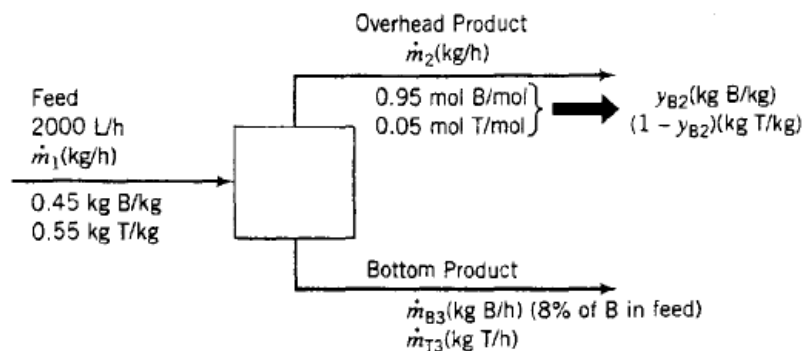
Solved examples in Material Balance Problems

Material Balances on a Distillation Column

A liquid mixture containing 45.0% benzene (B) and 55.0% toluene (T) by mass is fed to a distillation column. A product stream leaving the top of the column (the *overhead product*) contains 95.0 mole % B, and a bottom product stream contains 8.0% of the benzene fed to the column (meaning that 92% of the benzene leaves with the overhead product). The volumetric flow rate of the feed stream is 2000 L/h and the specific gravity of the feed mixture is 0.872. Determine the mass flow rate of the overhead product stream and the mass flow rate and composition (mass fractions) of the bottom product stream.

We will explicitly illustrate the implementation of the steps of the procedure just outlined.

- 1. Choose a basis.** Having no reason to do otherwise, we choose the given feed stream flow rate (2000 L/h) as the basis of calculation.
- 2. Draw and label the flowchart.**



Note several points about the flowchart labeling:

- A volumetric flow rate is given for the feed stream, but mass flow rates and fractions will be needed for balances. The mass flow rate of the stream should therefore be considered an unknown process variable and labeled as such on the chart. Its value will be determined from the known volumetric flow rate and density of the feed stream.
- Since mass balances will be written, the given component mole fractions in the overhead product stream will have to be converted to mass fractions. The mass fractions are accordingly labeled as unknowns.
- We could have labeled the mass flow rate and mass fractions of the bottom stream as we did the overhead. However, since we have no information about either the flow rate or composition of this stream, we have instead labeled the component flow rates (following the rule of thumb given in Step 2 of the general procedure).
- Every component mass flow rate in every process stream can be expressed in terms of labeled quantities and variables. (Verify this statement.) For example, the flow rates of toluene (kg T/h) in the feed, overhead, and bottom streams are, respectively, $0.55\dot{m}_1$, $\dot{m}_2(1 - y_{B2})$, and \dot{m}_{T3} . The flowchart is therefore labeled completely.
- The 8%–92% benzene split between the product streams is not a stream flow rate or composition variable; nevertheless, we write it on the chart to remind ourselves that it is an additional relation among the stream variables and so should be included in the degree-of-freedom analysis.

3. **Write expressions for the quantities requested in the problem statement.** In terms of the quantities labeled on the flowchart, the quantities to be determined are \dot{m}_2 (the overhead product mass flow rate), $\dot{m}_3 = \dot{m}_{B3} + \dot{m}_{T3}$ (the bottom product mass flow rate), $x_B = \dot{m}_{B3}/\dot{m}_3$ (the benzene mass fraction in the bottom product), and $x_T = 1 - x_B$ (the toluene mass fraction). Once we determine \dot{m}_2 , \dot{m}_{B3} , and \dot{m}_{T3} , the problem is essentially solved.

4. **Convert mixed units in overhead product stream** (see procedure preceding Example 3.3-3).

Basis: 100 kmol overhead \implies 95.0 kmol B, 5.00 kmol T

$$\implies (95.0 \text{ kmol B}) \times (78.11 \text{ kg B/kmol B}) = 7420 \text{ kg B}, \quad (5.00 \times 92.13) = 461 \text{ kg T}$$

$$\implies (7420 \text{ kg B}) + (461 \text{ kg T}) = 7881 \text{ kg mixture}$$

$$\implies y_{B2} = (7420 \text{ kg B}) / (7881 \text{ kg mixture}) = 0.942 \text{ kg B/kg} \quad (\text{write on chart})$$

The molecular weights of benzene (78.11) and toluene (92.13) were looked up in Table B.1.

5. **Perform degree-of-freedom analysis.**

- 4 unknowns ($\dot{m}_1, \dot{m}_2, \dot{m}_{B3}, \dot{m}_{T3}$)
- 2 material balances (since there are two molecular species in this nonreactive process)
- 1 density relationship (relating the mass flow rate to the given volumetric flow rate of the feed)
- 1 specified benzene split (8% in bottom-92% in overhead)

0 degrees of freedom

The problem is therefore solvable.

6. **Write system equations and outline a solution procedure.** The variables for which each equation will be solved are circled.

- **Volumetric flow rate conversion.** From the given specific gravity, the density of the feed stream is 0.872 kg/L. (*Verify.*) Therefore,

$$\dot{m}_1 = \left(2000 \frac{\text{L}}{\text{h}}\right) \left(0.872 \frac{\text{kg}}{\text{L}}\right)$$

- **Benzene split fraction.** The benzene in the bottom product stream is 8% of the benzene in the feed stream. This statement translates directly into the equation

$$\dot{m}_{B3} = 0.08(0.45\dot{m}_1)$$

There are two unknowns remaining on the flowchart (\dot{m}_2 and \dot{m}_{T3}), and we are allowed to write two balances. Balances on total mass and on toluene each involve both unknowns, but a benzene balance only involves \dot{m}_2 (convince yourself, remembering that \dot{m}_{B3} is now known), so we begin with that one.

- **Benzene balance** $0.45\dot{m}_1 = \dot{m}_2 y_{B2} + \dot{m}_{B3}$
- **Toluene balance** $0.55\dot{m}_1 = (1 - y_{B2})\dot{m}_2 + \dot{m}_{T3}$

7. **Do the algebra.** The four equations may be solved manually or with equation-solving software. If a manual solution is performed, each newly calculated variable value should be written on the flowchart for ease of reference in the remainder of the solution. The results are $\dot{m}_1 = 1744 \text{ kg/h}$, $\dot{m}_{B3} = 62.8 \text{ kg benzene/h}$, $\dot{m}_2 = 766 \text{ kg/h}$, and $\dot{m}_{T3} = 915 \text{ kg toluene/h}$. (*Verify.*) A total mass balance (which is the sum of the benzene and toluene balances) may be written as a check on this solution:

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_{B3} + \dot{m}_{T3} \implies 1744 \text{ kg/h} = (766 + 62.8 + 915) \text{ kg/h} = 1744 \text{ kg/h}$$

8. Calculate additional quantities requested in the problem statement.

$$\dot{m}_3 = \dot{m}_{B3} + \dot{m}_{T3} = 62.8 \text{ kg/h} + 915 \text{ kg/h} = \boxed{978 \text{ kg/h}}$$

$$y_{B3} = \frac{\dot{m}_{B3}}{\dot{m}_3} = \frac{62.8 \text{ kg B}}{978 \text{ kg/h}} = \boxed{0.064 \text{ kg B/kg}}$$

$$y_{T3} = 1 - y_{B3} = \boxed{0.936 \text{ kg T/kg}}$$

BALANCES ON MULTIPLE-UNIT PROCESSES

In the preceding sections, we referred rather loosely to “the system,” as in the statement “At steady state the rate at which benzene enters the system equals the rate at which it leaves.” Not much was said about what “the system” was. Up to now, however, not much needed to be said, since we have considered only processes involving a single unit—a mixer, or a distillation column, or a reactor—and that unit necessarily constituted the system.

Industrial chemical processes rarely involve just one process unit. One or more chemical reactors are often present, as are units for mixing reactants, blending products, heating and cooling process streams, separating products from each other and from unconsumed reactants, and removing potentially hazardous pollutants from streams prior to discharging the streams to the plant environment. Before we analyze such processes, we must take a closer look at what we mean by a system.

In general terms, a “system” is any portion of a process that can be enclosed within a hypothetical box (boundary). It may be the entire process, an interconnected combination of some of the process units, a single unit, or a point at which two or more process streams come together or one stream splits into branches. The inputs and outputs to a system are the process streams that intersect the system boundary.

A flowchart for a two-unit process is shown in Figure 4.4-1. Five boundaries drawn about portions of the process define systems on which balances may be written.

Boundary **A** encloses the entire process; the system defined by this boundary has as inputs Feed Streams 1, 2, and 3 and Product Streams 1, 2, and 3. (Convince yourself.) Balances on this system are referred to as **overall balances**. The stream that connects Units 1 and 2 is internal to this system and so would not enter into overall system balances.

Boundary **B** encloses a feed stream mixing point. Feed Streams 1 and 2 are inputs to this system and the stream flowing to Unit 1 is an output. Boundary **C** encloses Unit 1 (one input stream and two output streams), Boundary **D** encloses a stream splitting point (one input stream and two output streams), and Boundary **E** encloses Unit 2 (two input streams and one output stream).

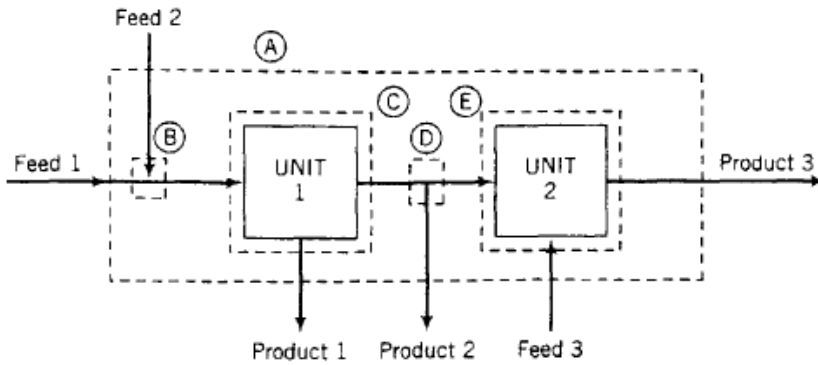
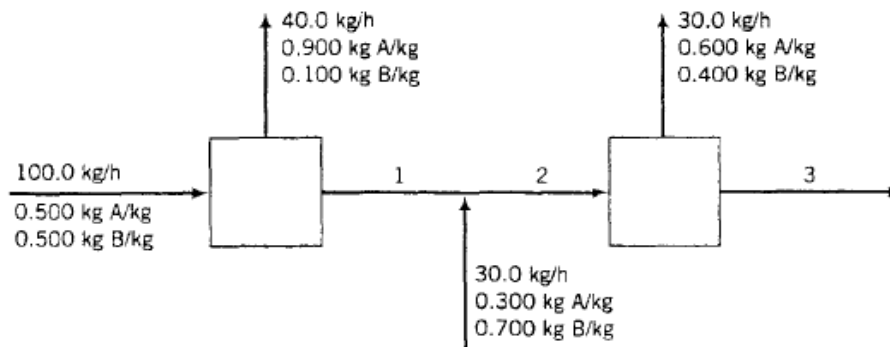


Figure 4.4-1 Flowchart of a two-unit process. Dashed lines denote boundaries of systems about which balances can be written.

The procedure for material balance calculations on multiple-unit processes is basically the same as that outlined in Section 4.3. The difference is that with multiple-unit processes you may have to isolate and write balances on several subsystems of the process to obtain enough equations to determine all unknown stream variables. When analyzing multiple-unit processes, carry out degree-of-freedom analyses on the overall process and on each subsystem, taking into account only the streams that intersect the boundary of the system under consideration. Do not begin to write and solve equations for a subsystem until you have verified that it has zero degrees of freedom.

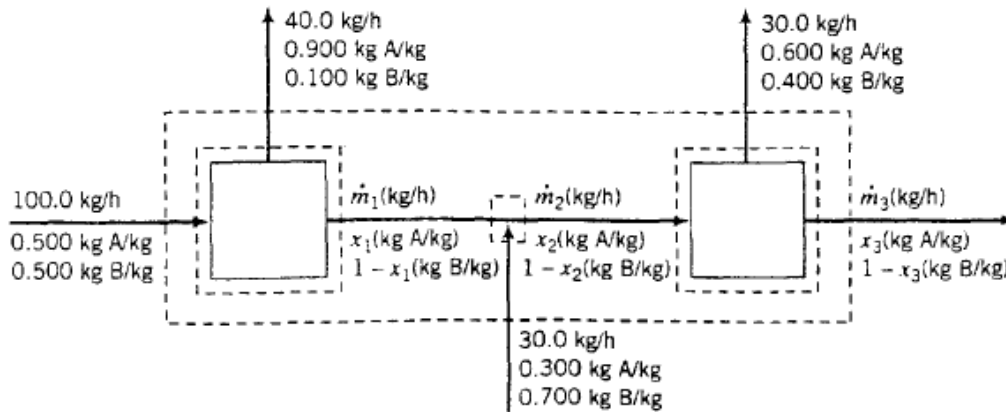
A labeled flowchart of a continuous steady-state two-unit process is shown below. Each stream contains two components, A and B, in different proportions. Three streams whose flow rates and/or compositions are not known are labeled 1, 2, and 3.



Calculate the unknown flow rates and compositions of streams 1, 2, and 3.

Basis—Given Flow Rates

The systems about which balances might be written are shown on the following representation of the flowchart:



The outer boundary encompasses the entire process and has as input and output streams all of the streams that enter and leave the process. Two of the interior boundaries surround individual process units, and the third encloses a stream junction point.

Degree-of-Freedom Analysis

We first outline the solution procedure by performing degree-of-freedom analyses on different systems. Remember that only variables associated with streams intersecting a system boundary are counted in the analysis of that system.

Overall system (outer dashed boundary):

$$2 \text{ unknowns } (\dot{m}_3, x_3) - 2 \text{ balances (2 species)} = 0 \text{ degrees of freedom}$$

$$\implies \underline{\text{Determine } \dot{m}_3 \text{ and } x_3}$$

In subsequent analyses, we may consider these two variables as known. Suppose we decide to consider the intermediate stream mixing point as the next system.

Mixing point:

$$4 \text{ unknowns } (\dot{m}_1, x_1, \dot{m}_2, x_2) - 2 \text{ balances (2 species)} = 2 \text{ degrees of freedom}$$

We have too many unknowns for the number of available equations. Let us try Unit 1.

Unit 1:

$$2 \text{ unknowns } (\dot{m}_1, x_1) - 2 \text{ balances (2 species)} = 0 \text{ degrees of freedom}$$

$$\implies \underline{\text{Determine } \dot{m}_1 \text{ and } x_1}$$

We may now analyze either the mixing point or Unit 2, each of which has two unknown variables associated with it.

Mixing point:

2 unknowns (\dot{m}_2, x_2) – 2 balances (2 species) = 0 degrees of freedom

⇒ Determine \dot{m}_2 and x_2

The procedure will therefore be to write overall system balances to determine \dot{m}_3 and x_3 , then balances on Unit 1 to determine \dot{m}_1 and x_1 , and finally balances on the intermediate mixing point to determine \dot{m}_2 and x_2 .

The calculations are straightforward. Note that all balances on this steady-state nonreactive process have the form *input* = *output*, and also note that the balances are written in an order that does not require solution of simultaneous equations (each equation involves only one unknown variable).

Calculations**Overall Mass Balance:**

$$(100.0 + 30.0) \frac{\text{kg}}{\text{h}} = (40.0 + 30.0) \frac{\text{kg}}{\text{h}} + \dot{m}_3 \implies \boxed{\dot{m}_3 = 60.0 \text{ kg/h}}$$

Overall Balance on A: (Verify that each additive term has the units kg A/h.)

$$(0.500)(100.0) + (0.300)(30.0) = (0.900)(40.0) + (0.600)(30.0) + x_3(60.0)$$

$$\implies \boxed{x_3 = 0.0833 \text{ kg A/kg}}$$

Mass Balance on Unit 1: (each term has the units kg/h)

$$100 = 40 + \dot{m}_1 \implies \boxed{\dot{m}_1 = 60.0 \text{ kg/h}}$$

A Balance on Unit 1: (each additive term has the units kg A/h)

$$(0.500)(100.0) = (0.900)(40.0) + x_1(60.0) \implies \boxed{x_1 = 0.233 \text{ kg A/kg}}$$

Mass Balance on Stream Mixing Point: (Each term has the units kg/h.)

$$\dot{m}_1 + 30.0 = \dot{m}_2 \xrightarrow{\dot{m}_1 = 60.0 \text{ kg/h}} \boxed{\dot{m}_2 = 90.0 \text{ kg/h}}$$

A Balance on Stream Mixing Point: (Each additive term has the units kg A/h.)

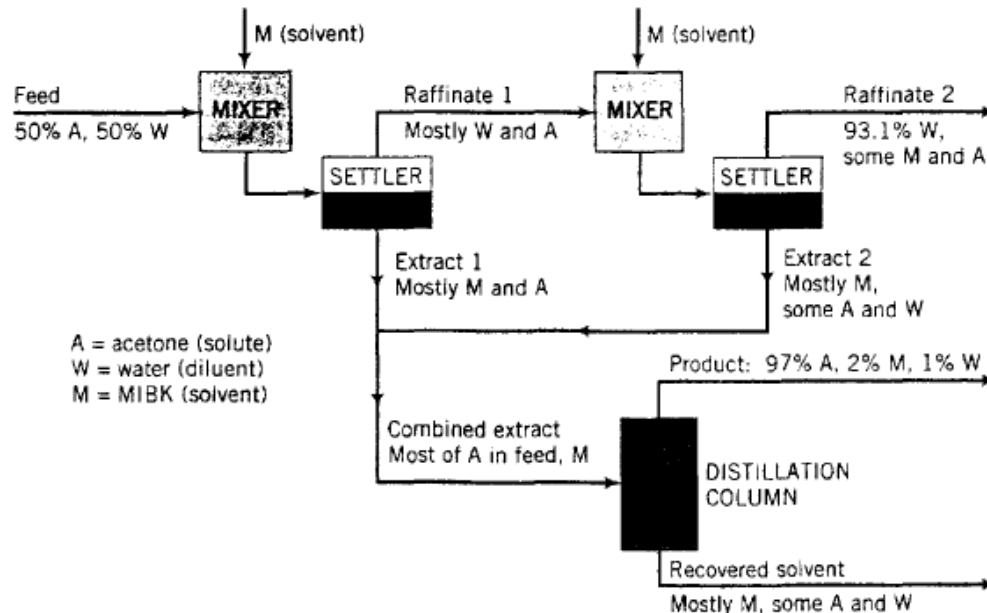
$$x_1 \dot{m}_1 + (0.300)(30.0) = x_2 \dot{m}_2$$

$$\begin{array}{l} \dot{m}_1 = 60.0 \text{ kg/h} \\ x_1 = 0.233 \text{ kg/kg} \\ \dot{m}_2 = 90.0 \text{ kg/h} \end{array}$$

$$\boxed{x_2 = 0.255 \text{ kg A/kg}}$$

An Extraction–Distillation Process

A mixture containing 50.0 wt% acetone and 50.0 wt% water is to be separated into two streams—one enriched in acetone, the other in water. The separation process consists of extraction of the acetone from the water into methyl isobutyl ketone (MIBK), which dissolves acetone but is nearly immiscible with water. The description that follows introduces some of the terms commonly used in reference to liquid extraction processes. The process is shown schematically below.



The acetone (**solute**)–water (**diluent**) mixture is first contacted with the MIBK (**solvent**) in a mixer that provides good contact between the two liquid phases. A portion of the acetone in the feed transfers from the aqueous (water) phase to the organic (MIBK) phase in this step. The mixture passes into a settling tank, where the phases separate and are separately withdrawn. The phase rich in the diluent (water, in this process) is referred to as the **raffinate**, and the phase rich in the solvent (MIBK) is the **extract**. The mixer–settler combination is the first **stage** of this separation process.

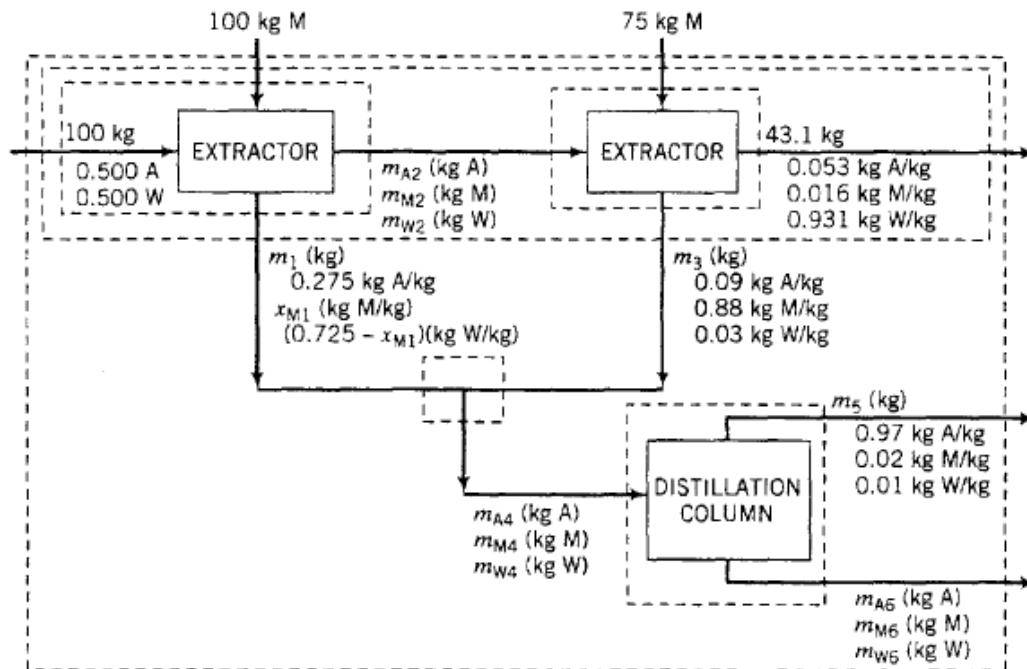
The raffinate passes to a second extraction stage where it is contacted with a second stream of pure MIBK, leading to the transfer of more acetone. The two phases are allowed to separate in a second settler, and the raffinate from this stage is discarded. The extracts from the two mixer–settler stages are combined and fed to a distillation column. The overhead effluent is rich in acetone and is the process product. The bottom effluent is rich in MIBK and in a real process would be treated further and recycled back to the first extraction stage, but we will not consider recycle in this example.

In a pilot–plant study, for every 100 kg of acetone–water fed to the first extraction stage, 100 kg of MIBK is fed to the first stage and 75 kg is fed to the second stage. The extract from the first stage is found to contain 27.5 wt% acetone. (All percentages in the remainder of this paragraph are weight percents.) The second-stage raffinate has a mass of 43.1 kg and contains 5.3% acetone, 1.6% MIBK, and 93.1% water, and the second-stage extract contains 9.0% acetone, 88.0% MIBK, and 3.0% water. The overhead product from the distillation column contains 2.0% MIBK, 1.0% water, and the balance acetone.

Taking a basis of calculation of 100 kg acetone–water feed, calculate the masses and compositions (component weight percentages) of the Stage 1 raffinate and extract, the Stage 2 extract, the combined extract, and the distillation overhead and bottoms products.

This is a “trick” problem in that there is not enough information to calculate all of the requested quantities. We will show how degree-of-freedom analysis allows a fairly quick determination of which variables can be determined and how to determine them efficiently, and also helps avoid wasting a great deal of time trying to solve a problem with insufficient information.

As always, we begin by drawing and labeling the flowchart. For simplicity, we will treat each mixer-settler combination as a single “extractor” unit.



Balances may be written for any of the systems shown on the chart, including the overall process, the individual extractors, the two-extractor combination, the point where the two extract streams combine, and the distillation column. A degree-of-freedom analysis for the overall process indicates that there are four unknown variables (m_5 , m_{A6} , m_{M6} , m_{W6}) and only three equations that relate them (one material balance for each of the three independent species involved in the process), leaving one degree of freedom. Similarly, the first extractor has two degrees of freedom, the second extractor has one, the extract mixing point has three, and the distillation column has four. (Verify these figures.) However, the system composed of the two extraction units involves only three unknowns (m_1 , x_{M1} , m_3) and three balances relating them, hence zero degrees of freedom. The solution procedure is as follows:

- **Analyze two-extractor subsystem.** Write a total mass balance and an acetone balance; solve simultaneously to determine m_1 and m_3 . Write an MIBK balance to determine x_{M1} .
- **Analyze extract mixing point.** Write acetone, MIBK, and water balances; solve to determine m_{A4} , m_{M4} , and m_{W4} , respectively.
- **Analyze first (or second) extractor.** Write acetone, MIBK, and water balances; solve to determine m_{A2} , m_{M2} , and m_{W2} .

At this point, we may quickly determine that we can go no farther. There are four remaining unknowns— m_5 , m_{A6} , m_{M6} , and m_{W6} . Whether we choose the overall process or the distillation col-

umn as our system, we will only have three independent equations and hence one degree of freedom, and so we will be unable to solve the problem. Moreover, since acetone, MIBK, and water all appear in both outlet streams, we cannot solve for any one of the individual unknowns. (If there were no water in the overhead product from the distillation column, for example, we could deduce that $m_{W6} = m_{W4}$). The problem is thus underspecified; unless another piece of information is furnished, the amounts and compositions of the distillation column products are indeterminate.

The calculations that can be performed are shown below. All balances have the form *input* = *output* (why?), and every additive term of each balance has units of kilograms of the balanced species.

Balances Around Two-Extractor Subsystem

$$\text{Total mass: } (100 + 100 + 75)\text{kg} = 43.1\text{ kg} + m_1 + m_3$$

$$\text{A: } 100(0.500)\text{ kg A} = (43.1)(0.053)\text{ kg A} + m_1(0.275) + m_3(0.09)$$

⇓ Solve simultaneously

$$m_1 = 145\text{ kg, } m_3 = 86.8\text{ kg}$$

$$\text{M: } (100 + 75)\text{ kg M} = (43.1)(0.016)\text{ kg M} + m_1 x_{M1} + m_3(0.88)$$

⇓ $m_1 = 145\text{ kg, } m_3 = 86.8\text{ kg}$

$$x_{M1} = 0.675\text{ kg MIBK/kg}$$

Balances Around Extract Mixing Point

$$\text{A: } m_1(0.275) + m_3(0.09) = m_{A4}$$

⇓ $m_1 = 145\text{ kg, } m_3 = 86.8\text{ kg}$

$$m_{A4} = 47.7\text{ kg acetone}$$

$$\text{M: } m_1 x_{M1} + m_3(0.88) = m_{M4}$$

$$\Downarrow m_1 = 145 \text{ kg, } m_3 = 87 \text{ kg, } x_{M1} = 0.675 \text{ kg M/kg}$$

$$m_{M4} = 174 \text{ kg MIBK}$$

$$\text{W: } m_1(0.725 - x_{M1}) + m_3(0.03) = m_{W4}$$

$$\Downarrow m_1 = 145 \text{ kg, } m_3 = 86.8 \text{ kg, } x_{M1} = 0.675 \text{ kg M/kg}$$

$$m_{W4} = 9.9 \text{ kg water}$$

Balances Around First Extractor

$$\text{A: } 100(0.500) \text{ kg A} = m_{A2} + m_1(0.275)$$

$$\Downarrow m_1 = 145 \text{ kg}$$

$$m_{A2} = 10.1 \text{ kg acetone}$$

$$\text{M: } 100 \text{ kg M} = m_{M2} + m_1 x_{M1}$$

$$\Downarrow m_1 = 145 \text{ kg, } x_{M1} = 0.675 \text{ kg M/kg}$$

$$m_{M2} = 2.3 \text{ kg MIBK}$$

$$\text{W: } (100)(0.500) = m_{W2} + m_1(0.725 - x_{M1})$$

$$\Downarrow m_1 = 145 \text{ kg, } x_{M1} = 0.675 \text{ kg M/kg}$$

$$m_{W2} = 42.6 \text{ kg water}$$

If we knew (or could independently determine) any one of the variables m_5 , m_{A6} , m_{M6} , or m_{W6} , we could calculate the remaining three. Since we do not, we must terminate the calculations at this point.

RECYCLE AND BYPASS

Material and Energy Balances on an Air Conditioner

Fresh air containing 4.00 mole% water vapor is to be cooled and dehumidified to a water content of 1.70 mole% H₂O. A stream of fresh air is combined with a recycle stream of previously dehumidified air and passed through the cooler. The blended stream entering the unit contains 2.30 mole% H₂O. In the air conditioner, some of the water in the feed stream is condensed and removed as liquid. A fraction of the dehumidified air leaving the cooler is recycled and the remainder is delivered to a

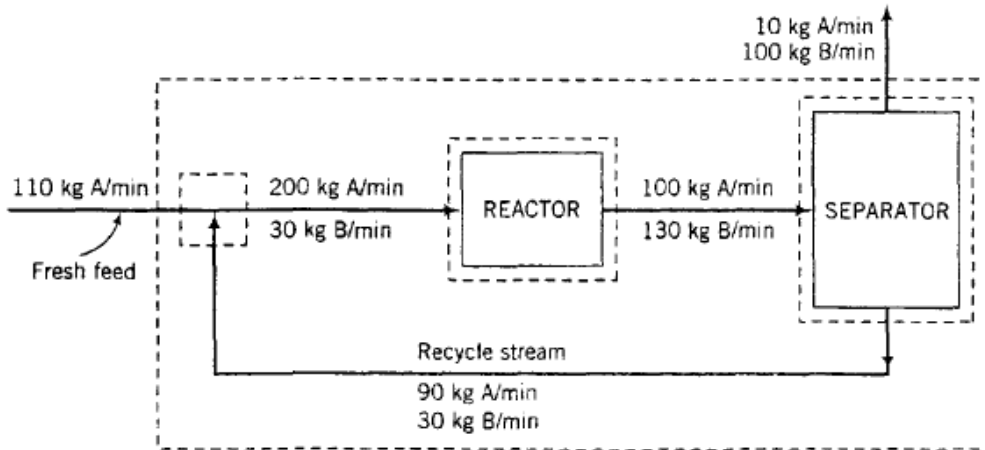
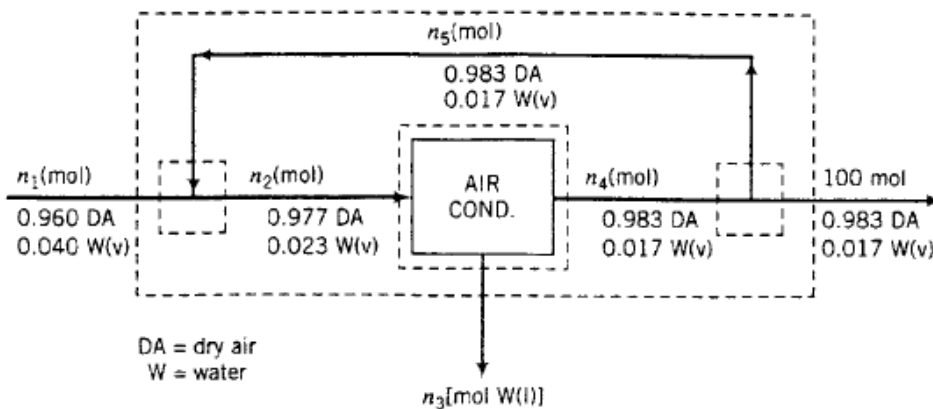


Figure 4.5-1 Flowchart of a reactor with separation and recycle of unconsumed reactant.

room. Taking 100 mol of dehumidified air delivered to the room as a basis of calculation, calculate the moles of fresh feed, moles of water condensed, and moles of dehumidified air recycled.

The labeled flowchart for this process, including the assumed basis of calculation, is shown below. Dashed lines depict the four subsystems about which balances might be written—the overall process, the recycle–fresh feed mixing point, the air conditioner, and the recycle–product gas splitting point. The quantities to be determined are n_1 , n_3 , and n_5 .



Students first attempting recycle problems are tempted to jump in and start writing balances immediately, and when they do they find themselves spending a *long* time on the problems, often failing to get solutions. If you first take a few minutes to go through a degree-of-freedom analysis you should be able to work out an efficient solution method before doing any calculations, ultimately saving yourself much more time than it took to do the analysis. Observe the procedure.

Overall System

2 variables (n_1, n_3)	(Only consider streams that intersect the system boundary.)
-2 balance equations	(Two species—dry air and water—are in the streams.)
0 degrees of freedom	

We can therefore determine n_1 and n_3 from overall system balances. We would write a dry air balance first since it only involves one unknown (n_1), while total mole and water balances involve both unknowns. Once n_1 has been determined, the second balance yields n_3 . No more can be done with the overall system, so we move on to other subsystems.

Mixing Point	Cooler	Splitting Point
2 variables (n_2, n_5)	2 variables (n_2, n_4)	2 variables (n_4, n_5)
-2 balances	-2 balances	-1 balance (see below)
0 degrees of freedom	0 degrees of freedom	1 degree of freedom

Only one independent balance can be written for the splitting point because the streams entering and leaving this subsystem are labeled as having identical compositions, so that the dry air/water mixture in all three streams behaves like a single species. (Convince yourself: write a total mole balance and a dry air or water balance around the splitting point and observe that you get the same equation each time.)

At this point we could either write balances around the mixing point to determine n_2 and n_5 or around the cooler to determine n_2 and n_4 (but not around the splitting point, which has one degree of freedom). The mixing point is the logical subsystem to attack, since the problem statement asks for n_5 but not n_4 . Writing and solving balances around the mixing point will therefore complete the solution.

Now, finally, we do the calculations. All balances have the form *input* = *output*, and each additive term in each equation has the units (mol of the balanced quantity).

$$\text{Overall dry air balance: } 0.960n_1 = 0.983(100 \text{ mol}) \implies n_1 = 102.4 \text{ mol fresh feed}$$

$$\text{Overall mole balance: } n_1 = n_3 + 100 \text{ mol} \xrightarrow{n_1 = 102.4 \text{ mol}} n_3 = 2.4 \text{ mol H}_2\text{O condensed}$$

$$\text{Mole balance on mixing point: } n_1 + n_5 = n_2$$

$$\text{Water balance on mixing point: } 0.04n_1 + 0.017n_5 = 0.023n_2$$

$$\begin{array}{c} \downarrow n_1 = 102.4 \text{ mol} \\ \downarrow \text{Solve simultaneously} \end{array}$$

$$n_2 = 392.5 \text{ mol}$$

$$n_5 = 290 \text{ mol recycled}$$

Almost three moles are recycled for every mole of air delivered to the room.

Reference:

Richard M. Felder, Ronald W. Rousseau, *Elementary Principles of Chemical Processes*, 3rd Ed., 2005, pp. 102 – 112.