**Dimensionless Numbers**

**Exercise 1**. Use of `input` and `disp` commands in Dimensionless number calculations. Create a script file incorporating the above-mentioned commands to calculate the Reynolds, Prandtl, Nusselt, Grashof, Schmidt and Archimedes number in U.S. customary units as well as in SI units.

a) Calculation of Reynolds Number

$$N_{Re} = \frac{DV\rho}{\mu}$$

U.S. customary units

$D = 3 \text{ in.} = 3/12 \text{ ft}$

$V = 6 \text{ ft/s}$

$\rho = 0.08 \text{ lbm/ft}^3$

$\mu = 0.015 \text{ cp} = (0.015)(0.000672) \text{ lbm/ft·s}$

$$N_{Re} = \frac{(3/12)(6)(0.08)}{(0.015)(0.000672)} = 11{,}904$$

SI units

$D = (3)(0.0254) \text{ m}$

$V = (6)(0.3048) \text{ m/s}$

$\rho = (0.08)(16.018) \text{ kg/m}^3$

$\mu = (0.015)(0.001) \text{ kg/m·s}$

$$N_{Re} = \frac{(3 \times 0.0254)\,(6 \times 0.3048)\,(0.08 \times 16.018)}{(0.015)\,(0.001)} = 11{,}904$$

Matlab Code using `input` and `disp` function. Open editor window and create the script file as given below:

U.S. Customary units

```
D   = input('Diameter,  (ft)              = ');
V   = input('Velocity,  (ft/s)            = ');
Rho = input('Density,   (lbm/cubic feet)  = ');
Mu  = input('Viscosity, (lbm/ft.s)        = ');
disp(' ')
disp('Reynolds no. = ')
disp(D*V*Rho/Mu)
```

Save the file as `reynoldsUS and` come to command window and execute the `reynoldsUS.m (m-file)` as given below:

```
>> reynoldsUS
Diameter,  (ft)                = 3/12
Velocity,  (ft/s)              = 6
Density,   (lbm/cubic feet)    = 0.08
Viscosity, (lbm/ft.s)          = 0.015*0.000672

Reynolds no. =
   1.1905e+04
```

SI Units

```
D   = input('Diameter,  (m)              = ');
V   = input('Velocity,  (m/s)            = ');
Rho = input('Density,   (kg/cubic meter) = ');
Mu  = input('Viscosity, (kg/m.s)         = ');
disp(' ')
disp('Reynolds no. = ')
disp(D*V*Rho/Mu)
```

Save the file as `reynoldsSI and` return to command window and execute the `reynolds.m (m-file)` as given below:

```
>> reynoldsSI
Diameter,  (m)              = 3*0.0254
Velocity,  (m/s)            = 6*0.3048
Density,   (kg/cubic meter) = 0.08*16.018
Viscosity, (kg/m.s)         = 0.015*0.001

Reynolds no. =
   1.1905e+04
```

Similary, try the following dimensionless numbers using the `input` and `disp` commands in MATLAB

b) Calculation of a Prandtl Number

$$N_{Pr} = \frac{C_p \mu}{k}$$

U.S. customary units

$\gamma_p = 0.47$ Btu/lbm °F

$\mu = 15$ centipoise $= (15)(0.000672)(3600)$ lbm/ft·hr

$k = 0.065$ Btu/hr·ft² (°F/ft)

$$N_{Pr} = \frac{(0.47)(15 \times 0.000672 \times 3600)}{0.065} = 262.4$$

SI units

$\gamma = (0.47)(4184)$ J/kg °C

$\mu = (15)(0.001)$ kg/m·s

$k = (0.065)(1.728)$ J/s·m² (°C/m)

$$N_{Pr} = \frac{(0.47)(4184)(15)(0.001)}{(0.065)(1.728)} = 262.6$$

c) Calculation of a Nusselt Number

$$N_{Nu} = \frac{hD}{k}$$

U.S. customary units

$h = 200$ Btu/hr·ft²·°F

$D = 1.5$ in. $= 1.5/12$ ft

$k = 0.07$ Btu/hr·ft² (°F/ft)

$$N_{Nu} = \frac{(200)(1.5/12)}{0.07} = 357.1$$

SI units

$h = (200)(5.678)$ J/(s·m²·°C)

$D = (1.5)(0.0254)$ m

$k = (0.07)(1.728)$ J/s·m² (°C/m)

$$N_{Nu} = \frac{(200)\,(5.678)\,(1.5)\,(0.0254)}{(0.07)\,(1.728)} = 357.7$$

d) Calculation of a Grashof Number

$$N_{Gr} = L^3\rho^2 g\beta(\Delta T)/\mu^2$$

U.S. Customary units

$L = 3$ ft
$\rho = 0.0725$ lbm/ft$^3$
$g = 32.174$ ft/s$^2$
$\beta = 0.00168/°R$
$\Delta T = 99$ °R
$\mu = 0.019$ centipoise $= 0.019 \times 0.000672$ lbm/ft·s
$= 1.277 \times 10^{-5}$ lbm/ft·s

$$N_{Gr} = \frac{(3^3)\,(0.0725)^2(32.174)\,(0.00168)\,(99)}{(1.277 \times 10^{-5})^2} = 4.66 \times 10^9$$

SI units

$L = (3)(0.3048) = 0.9144$ m
$\rho = (0.0725)(16.018) = 1.1613$ kg/m$^3$
$g = 9.807$ m/s$^2$
$\beta = (0.00168)/(1.8) = 0.000933/°K$
$\Delta T = (99)(1.8) = 178.2$ °K
$\mu = (0.019)(0.001) = 1.9 \times 10^{-5}$ kg/m·s

$$N_{Gr} = \frac{(0.9144)^3(1.1613)^2(9.807)(0.000933)(178.2)}{(1.9 \times 10^{-5})^2} = 4.66 \times 10^9$$

e) Calculation of a Schmidt Number

$$N_{Sc} = \frac{\mu}{\rho D}$$

U.S. customary units

$\mu = 0.02$ centipoise $= (0.02)(2.42)$ lbm/ft·hr
$\rho = 0.08$ lbm/ft$^3$
$D = 1.0$ ft$^2$/hr (diffusivity)

$$N_{Sc} = \frac{(0.02)\,(2.42)}{(0.08)(1.0)} = 0.605$$

SI units

$\mu = (0.02)(0.001)$ kg/m·s
$\rho = (0.08)(16.02)$ kg/m$^2$
$D = (1.0)(2.58 \times 10^{-5})$ m$^2$/s

$$N_{Sc} = \frac{(0.02)\,(0.001)}{(0.08)(16.02)(1.0)\,(2.58 \times 10^{-5})} = 0.605$$

f) Calculation of a Archimedes Number

$$N_{Ar} = \frac{d^3 \rho_f (\rho_p - \rho_f) g}{\mu^2}$$

U.S. customary units

$d = 2 \text{ mm} = 2/[(1000)(0.3048)] = 0.00656 \text{ ft}$

$\rho_f = 0.0175 \text{ lbm/ft}^3$

$\rho_p = 168.5 \text{ lbm/ft}^3$

$g = 32.174 \text{ ft/s}^2$

$\mu = 0.04 \text{ centipoise} = 0.04 \times 0.000672 = 2.688^{-5} \text{ lbm/ft·s}$

$$N_{Ar} = \frac{(0.00656)^3 (0.0175)(168.5 - 0.017)(32.174)}{(2.688 \times 10^{-5})^2} = 37{,}064$$

SI units

$d = 2/1000 \text{ m}$

$\rho_p = 168.5 \times 16.02 = 2699.37 \text{ kg/m}^3$

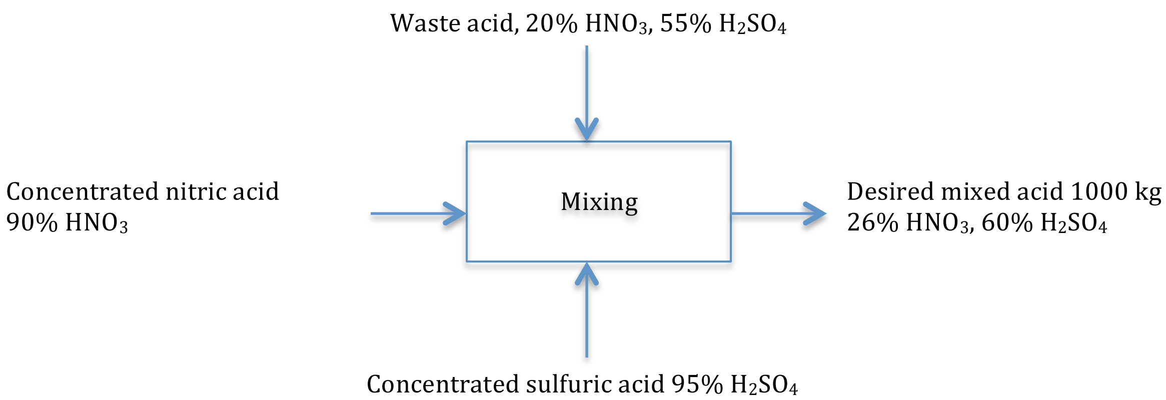
$\rho_f = 0.0175 \times 16.02 = 0.2804 \text{ g/m}^3$

$g = 9.807 \text{ m/s}^2$

$\mu = 0.04 \times 0.001 = 4 \times 10^{-5} \text{ kg/m·s}$

$$N_{Ar} = \frac{(2/1000)^3 (0.2804)(2699.37 - 0.28)(9.807)}{(4 \times 10^{-5})^2} = 37{,}118$$

**Exercise 2.** The waste acid from a nitrating process containing 20% $HNO_3$, 55% $H_2SO_4$ and 25% $H_2O$ by weight is to be concentrated by the addition of concentrated $H_2SO_4$ containing 95% $H_2SO_4$ and concentrated $HNO_3$ containing 90% $HNO_3$ to get desired mixed acid containing 26% $HNO_3$ and 60% $H_2SO_4$. Calculate the quantities of waste and concentrated acids required for 1000 kg of desired mixed acid.




By overall Balance;

$$x + y + z + = 1000$$

By $H_2SO_4$ balance;

$$0.55x + 0.95y = 600$$

By $HNO_3$ balance;

$$0.2x + 0.9z = 260$$

Solve the above material balance problem in MATLAB command window.

Solution:

Write the above equation in the matrix form as given below

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.57 & 0.95 & 0 \\ 0.2 & 0 & 0.9 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{bmatrix} 1000 \\ 600 \\ 260 \end{bmatrix}$$

Solution by MATLAB

```
>> A=[1 1 1; 0.55 0.95 0.0; 0.2 0 0.9];
>> B=[1000 600 260]';
>> X=A\B
X =

   400.0000
   400.0000
   200.0000
```

**Exercise 3.** Calculate the volume occupied by 0.2817 kmol of chlorine gas at a pressure of 100 kPa and 298 K.

**Solution**

By Ideal gas law, we know

$$PV = nRT$$

$$V = nRT/P$$

Therefore taking $n$ = 0.2817 kmol, $P$ = 100 kPa, $T$ = 298 K; $R$ = 8.31451 we have , $V$ = 6.98 cubic meter Using MATLAB Command window

```
>> P = 100;
>> n = 0.2817;
>> R = 8.31451;
>> T = 298;
>> V=n*R*T/100

V =
    6.9797
```

**Exercise 3** Create a script file to calculate the volume occupied by ideal gas for the exceise 2. Use `input` and `disp` commands as discussed in the calculation of dimensionless numbers of Exercise 1.

**Exercise 4** Find the specific volume of n-butane at 500 K and 18 atm using the Redlich-Kwong equation of state.

$$p = \frac{RT}{\hat{v} - b} - \frac{a}{\hat{v}(\hat{v} + b)}$$

$$\hat{v}^3(p) - \hat{v}^2(RT) + \hat{v}(a - pb^2 - RTb) - ab = 0$$

where

$$a = 0.42748\left(\frac{R^2T_c^2}{p_c}\right), \quad b = 0.08664\left(\frac{RT_c}{p_c}\right),$$

***Step 1*** First, you need to prepare an m-file (function file) that will calculate the $f(x)$, or here $f(v)$, given the temperature, pressure, and thermodynamic properties. The file is shown below.

```
function y=specvol(v)
Tc=425.2
pc=37.5
T=500
p=18
R=0.08206
aRK=0.42748*(R*Tc)^2/pc
aRK=aRK*(Tc/T)^0.5
bRK=0.08664*(R*Tc/pc)
y=p*v^3-R*T*v^2+(aRK-p*bRK^2-R*T*bRK)*v-aRK*bRK;
```

This function, called 'specvol', defines the problem you wish to solve.

***Step 2*** To test the function 'specvol' you issue either of the following commands:

```
feval('specvol',0.2)
ans=specvol(0.2)
```

The feval function causes MATLAB to compute the value of y (the output defined in specvol) using the m-file named specvol when $v = 0.2$. The output you get is:

```
Tc=425.2000
pc=37.5000
T=500
p=18
R=0.08206
```

```
aRK=13.8782
aRK=12.7981
bRK=0.0806
y=-0.6542
```

You should check these results line by line, especially the calculation of aRK, bRK, and y.

***Step 3*** When you use fzero, the function specvol will be evaluated for a variety of $v$. Thus, it is inconvenient to have the constants printed out on the screen every iteration. To avoid this, you change the function specvol by adding a semi-colon at the end of each line, ';'. This suppresses the output. Do this and save the m-file, specvol.

***Step 4*** Next you issue the command:

```
v=fzero('specvol',0.2)
v=2.0377
```

**Exercise 5.** Use `input, disp` and `fprintf` commands to create a script file for calculating "Diffusion of water through stagnant, non-diffusing air"

Water in the bottom of a narrow metal tube is held at constant temperature of 293 K. The total pressure of air (assumed dry) is $1.01325 \times 10^5$ Pa (1.0 atm) and the temperature is 293 K (20 $^{O}$C). Water evaporates and diffuses through the air in tube, and the diffusion path $Z_2$-$Z_1$ is 0.1524 m (0.5 ft) long. Calculate the rate of evaporation at steady state in kg mol/s · m$^2$. The diffusivity of water vapor at 293 K and 1 atm pressure is $0.250 \times 10^{-4}$ m$^2$/s. Assume the system is isothermal.

Data: $P_{BM} = 1.001\times 10^5$, $P_{A1} - P_{A2} = 2.341 \times 10^3$ Pa

**Solution**

$$N_A = \frac{D_{AB}P}{RT(z_2 - z_1)P_{BM}}(P_{A1} - P_{A2})$$

$$N_A = \frac{(0.250\times10^{-4})(1.01325\times10^5)(2.314\times10^3)}{8314(293)(0.1524)(1.001\times10^5)} = 1.595\times10^{-7}\,\text{kgmol/s}\cdot\text{m}^2$$

**MATLAB Code**

```
Dab    = input('Diffusivity,  (meter square/s)   = ');
P      = input('Total pressure,  (Pa)            = ');
Pa     = input('Differential pressure,   (Pa)    = ');
R      = input('Gas constant, (J/kmol K)         = ');
T      = input('Temperature, (K)                 = ');
Z      = input('Diffusion path, (meter square)   = ');
Pbm    = input('Log mean pressure, (Pa)          = ');
disp(' ')
disp('Rate of evaporation at steady state      = ');
disp(Dab*P*Pa/R*T*Z*Pbm)
fprintf('The rate of evaporation is %8.4f kg mol/s meter
```

```
square\n', Rate of evaporation at steady state)
```

Save the file in the name of rate and execute it from the command window

```
>> rate
Diffusivity,  (meter square/s)  = 0.250*10^-4
Total pressure,  (Pa)           = 1.01325*10^5
Differential pressure,   (Pa)   = 2.314*10^3
Gas constant, (J/kmol K)        = 8314
Temperature, (K)                = 273
Diffusion path, (meter square)  = 0.1524
Log mean pressure, (Pa)         = 1.001*10^5

Rate of evaporation at steady state      =
   2.9362e+06
```

**Exercise 6.** Use of `input, disp` and `fprintf` commands to create a script file to calculate the heat loss per meter square of surface area for an insulating wall composed of 25.4 mm thick fiber insulating board ($x_1$-$x_2$ = 0.0254 m), where the inside temperature is 352.7 K and the outside temperature is 297.1 K. The thermal conductivity of the fiber insulating board is 0.048 W/m·K

Solution

We know the basic equation to calculate the transfer of heat by conduction follows the relation

$$\frac{q}{A} = \frac{k}{x_1 - x_2}(T_1 - T_2)$$

$$\frac{q}{A} = \frac{0.048}{0.0254}(352.7 - 297.1) = 105.1 \text{ W/m}^2$$

**Exercise 7.** The temperature dependence of chemical reactions can be computed with the *Arrhenius equation:*

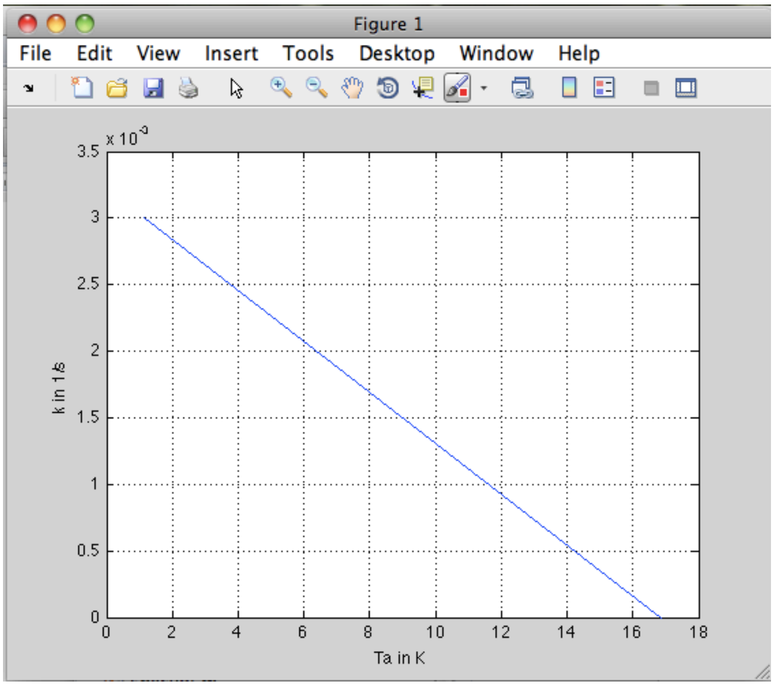
$$k = Ae^{-E/(RT_a)}$$

where $k$ = reaction rate (s-1), $A$ = preexponential factor (or frequency factor), $E$ = Activation energy (J/mol), $R$ = gas constant [8.314 J/mol . K], and $T_a$ = absolute temperature (K). A compound has $1 \times 10^5$ J/mol and $A = 7 \times 10^{16}$ J/mol. Use MATLAB command window to generate values of reaction rates ranging for temperature ranging from 273 to 333 K. Use plot to generate a graph of (a) $\log_{10} k$ versus $1/T_a$ and (b) employ `semilogx` function to create a plot for $\log_{10} k$ versus $1/T_a$

Solution

MATLAB code using command window

```
>> T=[273:10:333]'

T =

   273
   283
   293
   303
   313
   323
   333

>> length(T)

ans =

     7

>> A=(7)*(10^16);
>> E=1*10^5;
>> R=8.314;
>> x=(-10^5)/(8.314*T);
>> k=A*exp(x);
>> log10(k);
>> 1/T;
>> plot(log10(k),1/T);
>> xlabel('Ta in K')
>> ylabel('k in 1/s')
>> grid;
```




**Exercise 8.** It is general practice in engineering and science that equations be plotted as lines and discrete data as symbols. Here is some data for concentration ($c$) versus time ($t$) for the photodegradation of aqueous bromine

| ***t*, min** | **10** | **20** | **30** | **40** | **50** | **60** |
|---|---|---|---|---|---|---|
| ***c*, ppm** | 3.4 | 2.6 | 1.6 | 1.3 | 1.0 | 0.5 |

The above data can be described by the following function

$$c = 4.84e^{-0.034t}$$

Use MATLAB to create a plot displaying both the data (using circle 'o' symbol).

```
>> x=[10,20,30,40,50,60];
>> y=[2.4,2.6,1.6,1.3,1.0,0.5];
>> y=4.84*exp(-0.034*x);
>> plot(x,y,'o');
>> grid
>> xlabel('t, min');
>> ylabel('c, ppm');
```

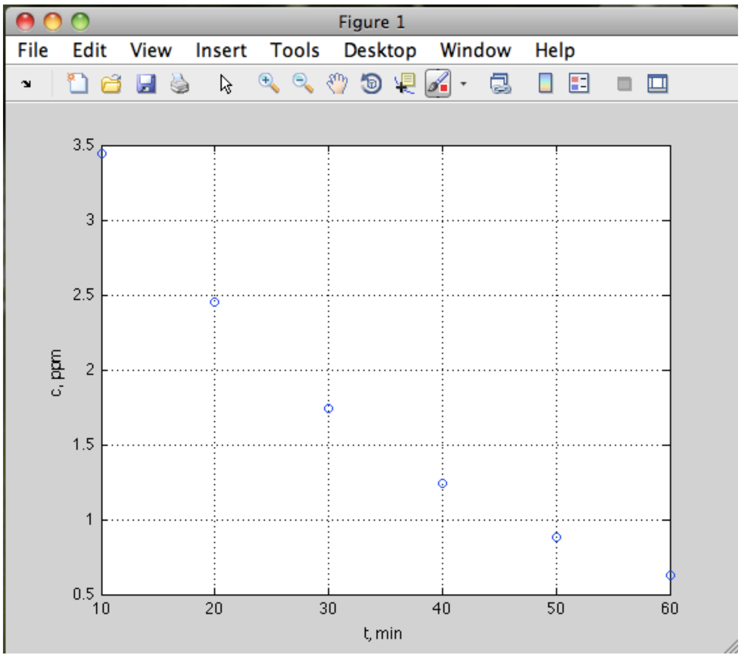


Figure 1
File Edit View Insert Tools Desktop Window Help
3.5
3
2.5
2
1.5
1
0.5
c, ppm
10
20
30
40
50
60
t, min