CH0401 Process Engineering Economics

Lecture 1c

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1. **Introduction – Time Value of Money**
2. Equivalence
3. Equations for economic studies
4. Amortization
5. Depreciation and Depletion

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S.No	Equation	Use
1.	$F = P(1+i)^n = PC_F$	Single payment at the end of n^{th} period
2.	$R = P\left(\dfrac{i(1+i)^n}{(1+i)^n - 1}\right) = \dfrac{P}{P_F}$	Uniform payment at the end of period (to pay fixed amount each year)
3.	$F = R\left(\dfrac{(1+i)^n - 1}{i}\right)$	Future worth at the end of n^{th} period
4.	$P = R\left(\dfrac{(1+i)^n - 1}{i(1+i)^n}\right) = RP_F$	Present Worth

S.No	Equation	Use
4.	$P = R\left(\frac{(1+i)^n - 1}{i(1+i)^n}\right) = RP_F$	Present Worth
5.	$R = (P-L)\left(\frac{i(1+i)^n}{(1+i)^n - 1}\right) + L \times i$	Uniform payment with salvage (*L*)
6.	$(1+i)^n = \frac{1}{1-\left(\frac{P}{R}\right)i}$	Rate of return or payment time when *L* is zero or salvage is neglected
7.	$n = \frac{-\log\left(1 - i\frac{P}{R}\right)}{\log(1+i)}$	Payment time when *L* is zero or salvage is neglected

S. No	Equation	Use
8.	$P' = \frac{R'}{i'}$	Capitalized costs (or) perpetual uniform payment R' to an equivalent capital cost P' at he present time for a given interest rate.
9.	$C_k = (C_{FC} - S_{FD}) f_k$ $\because f_k = \frac{(1+i)^n}{(1+i)^n - 1}$	Capitalized cost including cost factor.
10.	$R'' = P\left(\frac{i'}{(1+i')^n - 1}\right)$	Sinking fund deposit, i' – is sinking fund interest rate and L is zero.
11.	$P = R'''\left(\frac{(1+i')^n - 1}{i\left[(1+i')^n - 1\right] + i'}\right)$	Hoskold's formula - is rate of return, i' is sinking fund interest rate. Note that when $i = i'$ equation (10) reduces to equation (4)

i = interest rate per period

i' = sinking fund interest

P = present sum of money

F = sum at future date at 'n' Periods

R = end of period payment to give P in uniform series

L = salvage at some future date

C_F = compound interest factor equal to $(1+i)^n$

P_F = present worth factor equal to $\dfrac{(1+i)^n - 1}{i(1+i)^n} = \dfrac{P}{R}$

R'' = periodic sinking fund deposit R''

R''' = the annual payment R''' to the owners each year which pays them when the studies of capital recovery for exploitation of mineral resources.

C_{FC} = fixed capital cost of equipment for a finite life of 'n' years

C_k = capitalized cost of the equipment

S_{FD} = $\dfrac{S}{(1+i)^n}$; salvage value or scrap value with compound interest

f_k = capitalized cost factor

In the above table i.e. equations used for economic studies, the compound interest factors used in all the equations from 1 to ll are based on two series

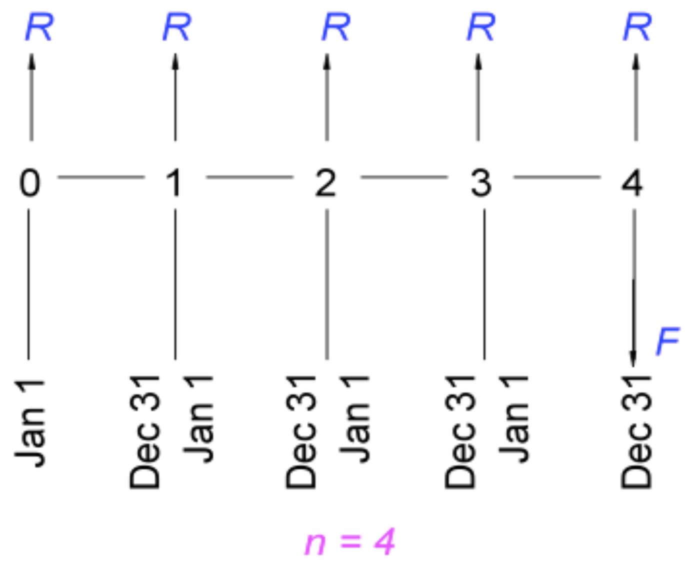
- Single Payment series
- Uniform annual series

Single Payment		Uniform annual series			
Compound-amount factor	Present-worth factor	Sinking-fund factor	Capital recovery factor	Compound-amount factor	Present-worth factor
Given P to Find F	Given F to Find P	Given F to Find R	Given P to Find R	Given R to Find F	Given R to Find P
$(1+i)^n$	$\frac{1}{(1+i)^n}$	$\frac{i'}{(1+i')^n-1}$	$\frac{i(1+i)^n}{(1+i)^n-1}$	$\frac{(1+i)^n-1}{i}$	$\frac{(1+i)^n-1}{i(1+i)^n}$

Interest formulas relating a uniform series to its present worth and future worth

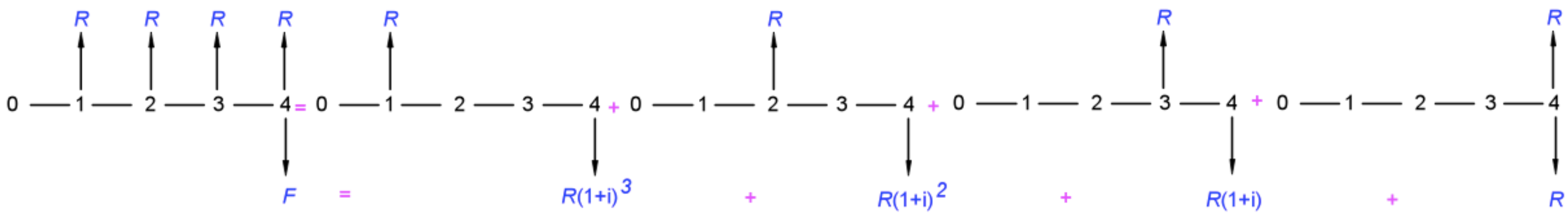
We will use the relationship $F = P(1+i)^n$ in our uniform series derivation

The general relationship between R and F is shown in the figure given below



Where R = An end of period uniform series for n periods
F = Future sum or Future worth

Looking at the figure given below we see that if amount *R* is invested at end of each year for 4 years, the total amount *F* at the end of 4 years will be the sum of the compound amounts of the individual investments



In general case for *n* years

$$F = R(1+i)^{n-1} + \dots\dots + R(1+i)^3 + R(1+i)^2 + R(1+i) + R \quad \dots\dots\dots\dots (1)$$

Where *R* = An end of period uniform series for *n* periods
F = Future sum or Future worth

Multiplying equation (1) by (1+i), we have

$$(1+i)F = R(1+i)^n + \cdots\cdots + R(1+i)^4 + R(1+i)^3 + R(1+i)^2 + R(1+i) \quad \cdots\cdots \quad (2)$$

Factoring out *R* and subtracting equation (1) gives

$$(1+i)F = R[(1+i)^n + \cdots\cdots + (1+i)^4 + (1+i)^3 + (1+i)^2 + (1+i)] \quad \cdots\cdots \quad (3)$$

$$-\quad F = R[(1+i)^{n-1} + \cdots\cdots + (1+i)^3 + (1+i)^2 + (1+i) + 1] \quad \cdots\cdots \quad (4)$$

$$iF = R[(1+i)^n - 1]$$

Solving above equation $iF = R[(1+i)^n - 1]$ for *F* gives

$$F = R\left[\frac{(1+i)^n - 1}{i}\right] \quad ----(5)$$

Thus we have an equation for F when R known i.e

$$F = R\left[\frac{(1+i)^n - 1}{i}\right] \quad ----(5)$$

The term inside the brackets

$$\left[\frac{(1+i)^n - 1}{i}\right]$$

is called uniform series compound amount factor

We know that $F = P(1+i)^n$ Substituting this equation for F in equation (5) we get

$$F = R\left[\frac{(1+i)^n - 1}{i}\right] \quad -----(5)$$

$$P(1+i)^n = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \quad -----(6)$$

Above equation (6) takes the form of ***equation no. 4*** of equations for economic studies given in the table (slide no. 6). The equation (6) can be used to calculate P if R is known. **(Nomenclature for the above equations are given in slide no. 8)**

$$F = R\left[\frac{(1+i)^n - 1}{i}\right] \quad -----(5)$$

Above equation (5) takes the form of ***equation no. 3*** of equations for economic studies given in the table (slide no. 5). The equation (5) can be used to calculate F if R is known. **(Nomenclature for the above equations are given in slide no. 8)**

We know that

$$P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \quad -----(6)$$

Rearranging the above equation (6), we have

$$\frac{P}{R} = \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$R = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \quad ---- (7)$$

$$R = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \text{ ---- (7)}$$

Above equation (7) takes the form of ***equation no. 2*** of equations for economic studies given in the table (slide no. 5). The equation (7) can be used to calculate R if P is known. **(Nomenclature for the above equations are given in slide no. 8)**

Above equation (7) or ***equation no. 2*** of equations for economic studies given in the table (slide no. 5) can be re arranged as follows .

$$R = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \quad \text{---- (7)}$$

$$R[(1+i)^n - 1] = Pi(1+i)^n$$

$$[(1+i)^n - 1] = \frac{Pi}{R}[(1+i)^n]$$

$$\frac{[(1+i)^n - 1]}{(1+i)^n} = \frac{Pi}{R}$$

$$\frac{(1+i)^n}{(1+i)^n} - \frac{1}{(1+i)^n} = \frac{Pi}{R}$$

$$1 = \frac{Pi}{R}$$

$$1 = \frac{Pi}{R} + \frac{1}{(1+i)^n}$$

$$1 - \frac{Pi}{R} = \frac{1}{(1+i)^n}$$

$$1 - \frac{Pi}{R} = \frac{1}{(1+i)^n}$$

$$\frac{1}{1 - \frac{Pi}{R}} = (1+i)^n$$

$$i.e.\ (1+i)^n = \frac{1}{1 - \frac{Pi}{R}} \quad \text{-----(8)}$$

$$(1+i)^n = \frac{1}{1-\frac{Pi}{R}} \quad \text{-----(8)}$$

Above equation (8) takes the form of ***equation no. 6*** of equations for economic studies given in the table (slide no. 6). The equation (8) can be used to calculate *rate of return* or *Payment time when L is zero* or *salvage/scrap value is* neglected. **(Nomenclature for the above equations are given in slide no. 8)**

Taking log on both sides of equation (8) we have

$$n\log(1+i) = \log(1) - \log\left(1-\frac{Pi}{R}\right)$$

$$n = \frac{-\log\left(1-\frac{Pi}{R}\right)}{\log(1+i)} \quad \text{----- (9)}$$

$$n\log(1+i) = \log(1) - \log\left(1 - \frac{Pi}{R}\right)$$

$$n = \frac{-\log\left(1 - \frac{Pi}{R}\right)}{\log(1+i)} \text{ ----- (9)}$$

Above equation (8) takes the form of ***equation no. 6*** of equations for economic studies given in the table (slide no. 6). The equation (8) can be used to calculate *rate of return* or *Payment time when L is zero* or *salvage/scrap value* is neglected. **(Nomenclature for the above equations are given in slide no. 8)**

Process Engineering Economics – ***References***

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