CH0401 Process Engineering Economics

Lecture 1a

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1. **Introduction – Time Value of Money**
2. Equivalence
3. Equations for economic studies
4. Amortization
5. Depreciation and Depletion

Simple Interest

Simple interest is interest that is computed only on the original sum and not on accrued interest. Thus if you lend loan, a present sum of money P to someone at a simple annual interest rate i (stated as a decimal) for a period of n years, the amount of interest you would receive from the loan would be:

$$\text{Total interest earned} = P \times i \times n = Pin$$

At the end of n years the amount of money due you, *F-Futrue worth,* would equal the amount of the loan. P plus the total interest earned. That is, the amount of money due at the end of the loan would be:

$$F = P + Pin$$

$$\text{i.e. } F = P(1+in)$$

1. **Introduction – Time Value of Money**
2. Equivalence
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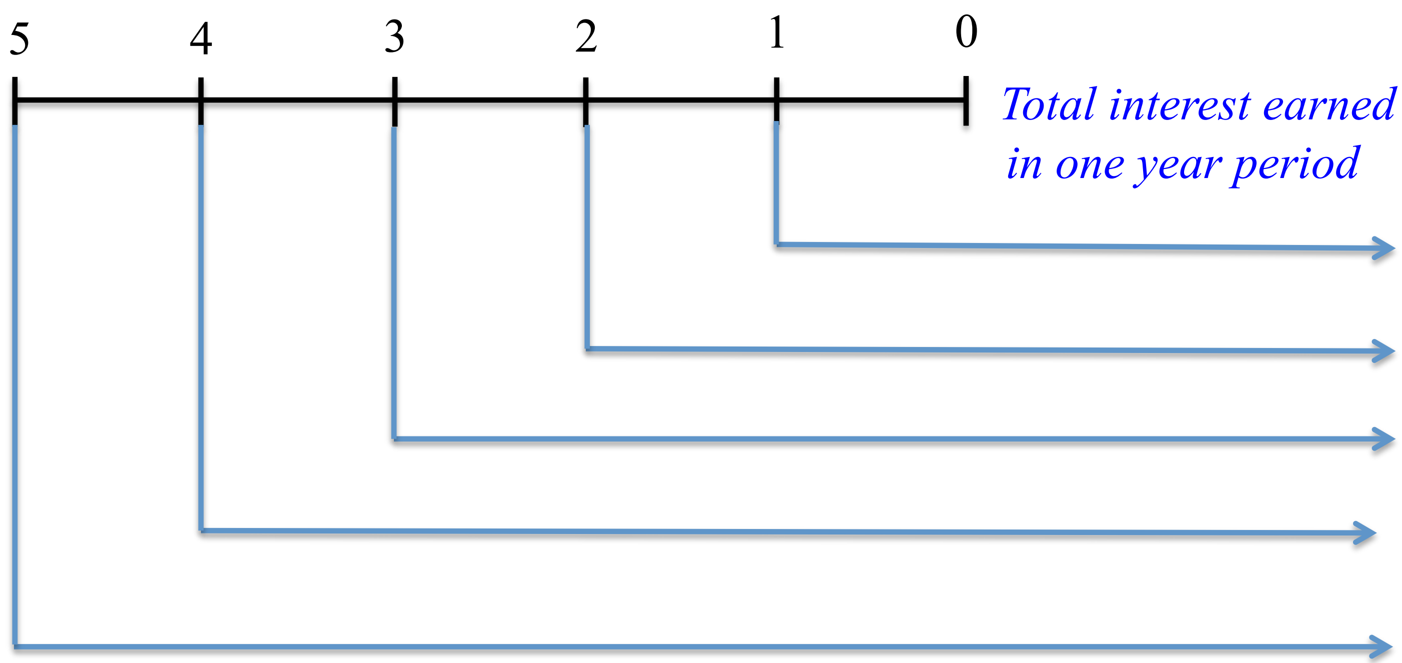
You have agreed to loan a friend ₹5000 for 5 years at a simple interest rate of 8% per year. How much interest will you receive from the loan? How much will your friend pay you at the end of 5 years?

P = ₹5000.00, $i = 0.08$, $n = 5$ years

Solution



Total interest earned or recieved in one year period = Pin = 400.00

= 400.00

= 400.00

= 400.00

= 400.00

= 5400.00

At the end of 5th year $F = P + Pin = P(1+in)$ = ₹7000.00

Process Engineering Economics – ***Simple Interest***

Contd……..

You have agreed to loan a friend ₹5000 for 5 years at a simple interest rate of 8% per year. How much interest will you receive from the loan? How much will your friend pay you at the end of 5 years?

Solution contd….

It clear form the solution seen in previous slide that the *interest received* at the end of each year is ₹400 and after 5 years it would be ₹2000.

Therefore at the end of 5^{th} year your friend would pay *interest plus the principal* that is 5000 + 2000 = ₹7000

Compound Interest — The interest earned on accumulated, reinvested interest as well as principal amount.

For a loan, any interest owed but not paid at the end of the year is added to the balance due. Thus, the next year's interest is calculated based on the unpaid balance due, which includes the unpaid interest from the preceding period. In this way, compound interest can be thought of as *interest on top of interest*. This distinguishes compound interest from simple interest

Let i = interest rate per interest period. In the equations the interest rate is stated as a decimal(that is, 9% interest is 0.09).

n = number of interest periods.

P = a present sum of money.

F = a future sum of money.

The future sum *F* is an amount, *n* interest periods from the present, that is equivalent to *P* with interest rate *i*.

Contd….

Suppose a present sum of money P is invested for one year at interest rate i.

At the end of the year, we should receive back our initial investment P, together with interest equal to iP, or a total amount $P + iP$.

Factoring P, the sum at the end of one year is $P(1+ i)$.

Let us assume that, instead of removing our investment at the end of one year, we agree to let it remain for another year. How much would our investment be worth at the end of the second year? The end-of-first-year sum $P(1 + i)$ will draw interest in the second year of $iP(1 + i)$.

This means that, at the end of the second year, the total investment will become

$$P(1+i) + iP(1+i)$$

This may be rearranged by factoring $P(1 + i)$, which gives us

$$P(1 + i)(1 + i) \quad \text{(Or)}$$

$$P(1 + i)^2$$

Contd….

If the process is continued for a third year, the end-of-the-third-year total amount will be $P(1 + i)^3$; at the end of n years, we will have $P(1 + i)^n$. The progression looks like this:

Year	Amount at the Beginning of Interest Period	+ Interest for Period	= Amount at End of Interest Period
1	P	$+ iP$	$= P(1+i)$
2	$P(1 + i)$	$+ iP(1+i)$	$= P(1+i)^2$
3	$P(1 + i)^2$	$+ iP(1+i)^2$	$= P(1 + i)^3$
n	$P(1+i)^{n-1}$	$+ iP(1+i)^{n-1}$	$=P(1+i)^n$

In other words, a present sum P increases in n periods to $P(1 + i)^n$.

Contd….

We therefore have a relationship between a present sum *P* and its equivalent future sum, *F*.

$$Future\ sum = (Present\ sum)\ (1 + i)^{n}$$

$$i.e.\quad F = P\,(1 + i)^{n}$$

This is the *single payment compound amount formula*

In the following table we calculate on a year-to-year basis the total amount due at the end of each year. Notice that this amount becomes the principal upon which interest is calculated in the next year (this is the compounding effect).

Contd….

Year	Total Principal (*P*) on Which Interest is Calculated in Year *n*	Interest (*i*) Owed at the End of Year n from Year n's Unpaid Total Principal	Total Amount Due at the End of Year *n*, New Total Principal for Year *n* + 1
1	**₹5000**	5000 x 0.08 = 400	5000 + 400 = 5400
2	5400	5400 x 0.08 = 432	5400 + 432 = 5832
3	5832	5832 x 0.08 = 467	5832 + 467 = 6299
4	6299	6299 x 0.08 = 504	6299 + 504 = 6803
5	6803	6803 x 0.08 = 544	6803 + 544 = **₹7347**

The total amount due at the end of the fifth year, ₹7347, is the amount that your friend will give you to repay the original loan. Notice that this amount is ₹347 more than the amount you received for loaning the same amount, for the same period, at simple interest. This, of course, is because of the effect of interest being earned (by you) on top of interest.

Suppose the bank changed their interest policy to *6% interest, compounded quarterly*. For this situation, how much money would be in the account at the end of 3 years, assuming a ₹500 deposit now?

Solution

First, we must be certain to understand the meaning of 6% *interest, compounded quarterly.* There are two elements:

6% interest: Unless otherwise described, it is customary to assume that the stated interest is for a one-year *period. If the stated interest is for other than a one-year period, the time frame must be clearly stated.*

Compounded quarterly: This indicates there are four interest periods per year; that is, an interest period is 3 months long.

We know that the 6% interest is an annual rate because if it were for a different period, it would have been stated. Since we are dealing with four interest periods per year, it follows that the interest rate per interest period is 1.5% . For the total 3-year duration, there are 12 interest periods.

Contd….

Suppose the bank changed their interest policy to *6% interest, compounded quarterly*. For this situation, how much money would be in the account at the end of 3 years, assuming a ₹500 deposit now?

Solution Thus

P = ₹500 $i = 0.015$ $n = (4 \times 3) = 12$ F = unknown

$$F = P(1+i)^n = ₹500\,(1+0.015)^{12} = ₹500(1.196) = ₹597.8$$

A $500 deposit now would yield ₹597.8 in 3 years.

P = $500 $i = 0.06$ $n = 3$ F = unknown

$$F = P\left(1+\frac{i}{m}\right)^{mn} = ₹500\,(1+0.015)^{12} = ₹500(1.196) = ₹597.8$$

A ₹500 deposit now would yield $598 in 3 years.

Where m is the number of times interest compounded in a year

Effective interest rate

Assume CITY BANK, India offers a nominal interest rate of 4% (i = 0.04) on your savings deposit. The table given below will illustrate the different effective or true interest rate depends on how many times the interest is compounded in a year.

S. No	Compounding	Formula	Effective interest rate
1	Annually	$i_{eff} = \left(1+\frac{0.04}{1}\right)^{1} - 1$	4.00%
2	Semi Annually	$i_{eff} = \left(1+\frac{0.04}{2}\right)^{2\times 1} - 1$	4.04%
3	Quaterly	$i_{eff} = \left(1+\frac{0.04}{4}\right)^{4\times 1} - 1$	4.06%
4	Monthly	$i_{eff} = \left(1+\frac{0.04}{12}\right)^{12\times 1} - 1$	4.07%
5	Weekly	$i_{eff} = \left(1+\frac{0.04}{52}\right)^{52\times 1} - 1$	4.08%
6	Daily	$i_{eff} = \left(1+\frac{0.04}{365}\right)^{365\times 1} - 1$	4.08%
7	Hourly	$i_{eff} = \left(1+\frac{0.04}{8760}\right)^{8760\times 1} - 1$	4.08%
8	Continuously	$i_{eff} = e^{0.04} - 1$	4.08%

Conservation of Money in a Bank Account

Banks lend money at compound intresest, and the simplest way of understanding the changing balance in the account is to assume that money in a bank account is a conserved quantity. That is, the money deposited (input) minus the money withdrawn (output) must be equal to the amount of money that accumulates. Thus, the conservation of money in a bank account can be treated as just as the conservation of mass. Of course, principle is valid only for bank accounts and not for the federal government, because the government can simply print additional money. However, recognizing this restriction, we can write

$$Accumulation = Input - Output \text{ ----- (1)}$$

Continuous interest

Some banks are now offering continuous compounding on money, rather than compounding the interest at discrete intervals. Since the continuous compounding case is similar to other conservation problems that chemical engineers study, we consider it first.

We let $P|_t$ be the money we have in the bank at time *t*, if we make no deposits or withdrawls, the amount we have in the bank will increase to $P|_{t+\Delta t}$ over a time interval Δt because the bank pays us the interest. If we let the continuous interest rate be i_c [$ interest/($ in account) (yr)], then the amount bank pays us in the time interval Δt is i_c $P|_t \Delta t$.

Hence the conservation equation becomes

Output - Input = Accumulation ----- (2)

Contd……

i.e., $P|_{t+\Delta t} - P|_t = i_c P|_t \Delta t$ ------ (3)

Now if we divide the equation (3) by Δt

$$\frac{P|_{t+\Delta t} - P|_{\Delta t}}{\Delta t} = \frac{i_c P|_t \Delta t}{\Delta t}$$

And taking limit as Δt approaches zero, we obtain

$$\underset{\Delta \to 0}{Limit} \frac{P(t+\Delta t) - P(t)}{\Delta t} = \frac{dP}{dt} = i_c P$$

i.e. $\dfrac{dP}{P} = i_c dt$

Integrating the above equation we have

$$\int \frac{dP}{P} = i_c \int dt$$

i.e. $\ln(P) = i_c t + C$ therefore,;

$$P = e^{i_c t} . e^c$$

Contd….

When t = 0, $F = P$

$$P = e^{0} . e^{c}$$

$$P = 1e^{c}$$

substituting $P = e^{c}$ in above equation $P = e^{i_c t} . e^{c}$

we have $F = P \times e^{i_c t}$

where F = future sum of money

P = Present sum of money

i_c = continuous interest

$t = n$ = number of years

Process Engineering Economics – Summary

Interest rates

Simple Interest

$$F = P(1+in)$$

Interest that is computed only on the original sum and not on accrued interest.

Compound Interest

$$F = P(1+i)^{n}$$

The interest earned on accumulated, reinvested interest as well as principal amount.

Nominal Interest

The annual interest rate without considering the effect of any compounding.

Effective Interest

$$F = P\left(1+\frac{i}{m}\right)^{mn}$$

The annual interest rate taking into account the effect of any compounding during the year.

Process Engineering Economics – Summary

Interest rates

- **Simple Interest**
  Interest that is computed only on the original sum and not on accrued interest.
- **Compound Interest**
  The interest earned on accumulated, reinvested interest as well as principal amount.
- **Nominal Interest**
  The annual interest rate without considering the effect of any compounding.
- **Effective Interest**
  The annual interest rate taking into account the effect of any compounding during the year.

Process Engineering Economics – ***References***

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