Computer Aided Process Plant Design

Material Balance

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OUTLINE

1. Material Balance
2. Classification
3. Algorithm
4. Example Problem

1. MATERIAL BALANCE

Law of Conservation of Mass

- Matter (Solid, Liquid and Gases) can take one form into other but the total amount of mass remains unchanged.
- “Mass is neither created nor destroyed”
- Father of Modern Chemistry (a French Nobel man)
- Antoine-Laurent de Lavoisier, 1789

1. MATERIAL BALANCE

Law of Conservation of Mass

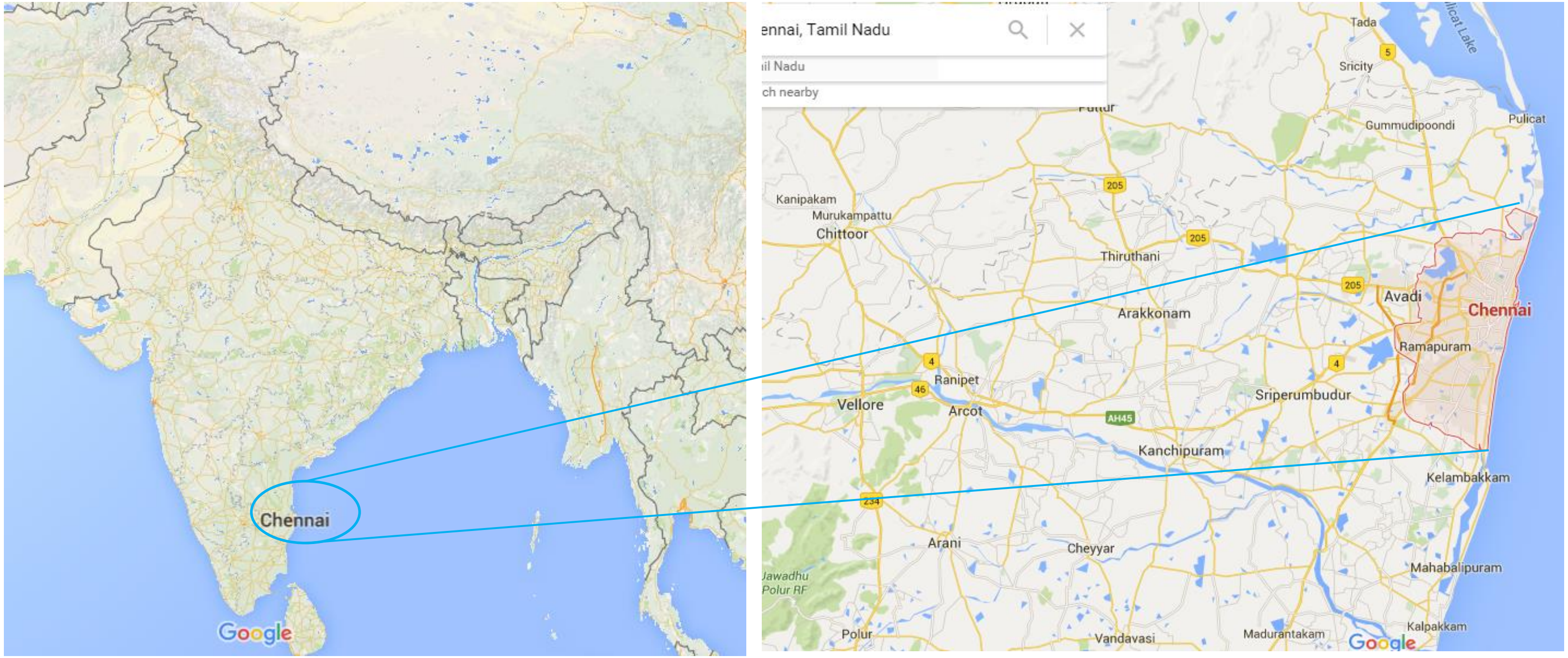
A balance on a conserved quantity (total mass, mass of particular component (or species), energy, momentum) in a system (a single process unit, a collections of units or an entire process) may be written in the following way:

Input	+	Generation	-	Output	-	Consumption	=	Accumulation
(enters through system boundaries)		(produced within system)		(leaves through the system boundaries)		(consumed within the system)		(build up within the system

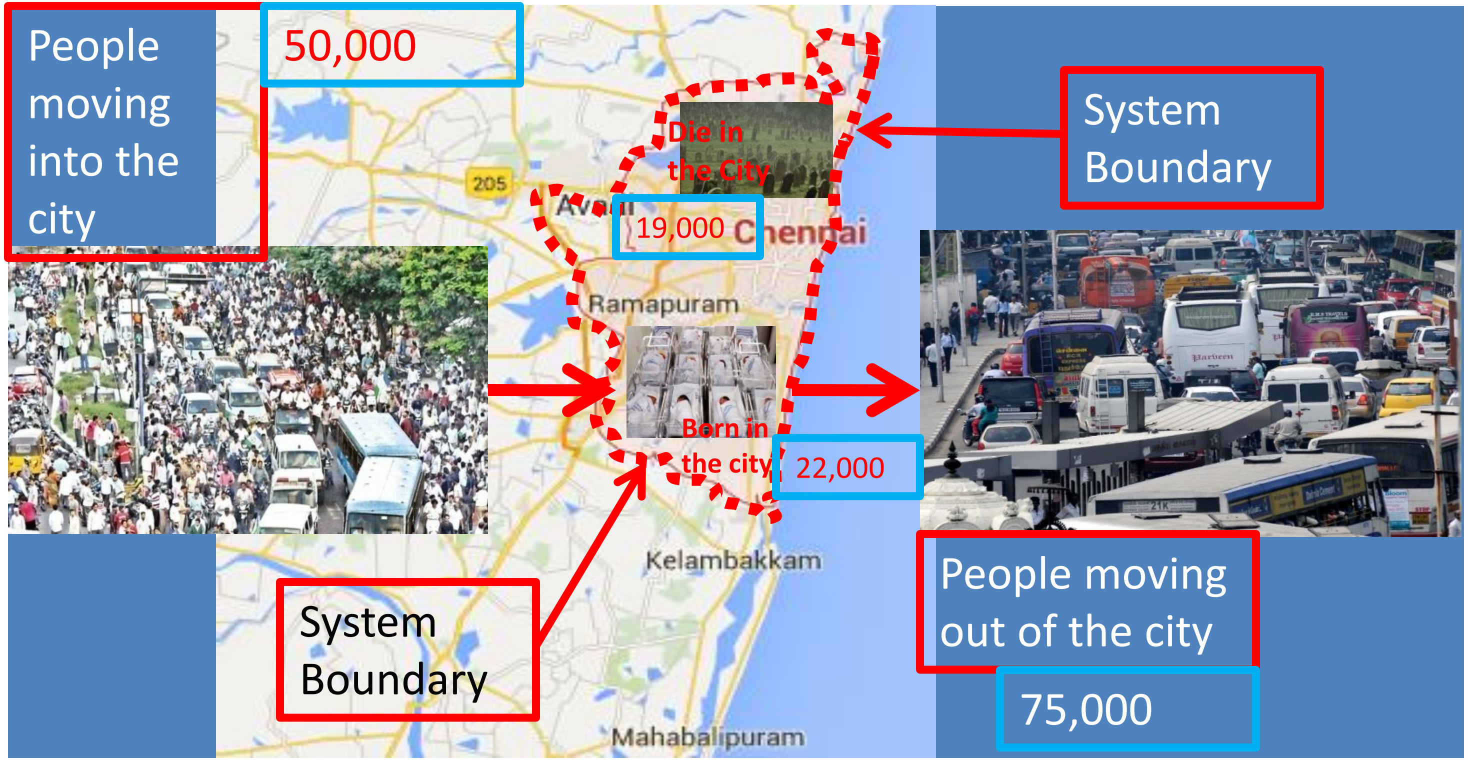
Law of Conservation of Mass - Example

Each year 50,000 people move into the a city like Chennai, Tamil Nadu, India.75,000 people move out, 22,000 are born, and 19,000 die. Write a balance on the population of the city

Law of Conservation of Mass - Example

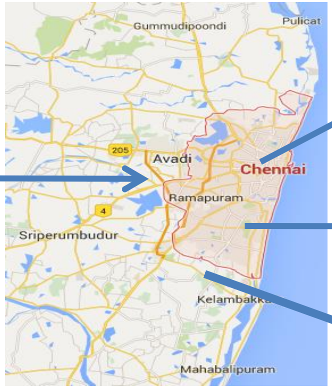


Law of Conservation of Mass - Example



Law of Conservation of Mass - Example

People move into the city, 50,000 P/yr



Born in the City, 22,000 P/yr

People move out of the city, 75,000 P/yr

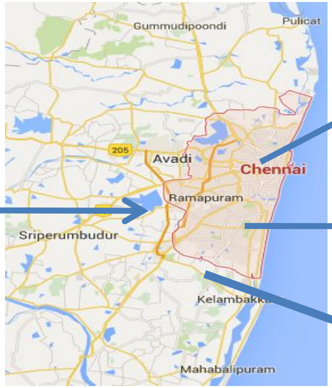
Die in the City, 19,000 P/yr

Balance equation

Input	+	Generation	-	Output	-	Consumption	=	Accumulation
50,000 (enters through system boundaries)		22,000 (produced within system)		75,000 (leaves through the system boundaries)		19,000 (consumed within the system)		= ? (build up within the system

Law of Conservation of Mass - Example

People move into the city, 50,000 P/yr



Born in the City, 22,000 P/yr

People move out of the city, 75,000 P/yr

Die in the City, 19,000 P/yr

Each year city population decreases by 22,000 P/yr

Balance equation

Input	+	Generation	-	Output	-	Consumption	=	Accumulation
50,000 (enters through system boundaries)		22,000 (produced within system)		75,000 (leaves through the system boundaries)		19,000 (consumed within the system)		= -22,000 (build up within the system

Classification of Material Balance Problems

Without Chemical reaction

- Batch
- Continuous
- Semi-continuous
- Steady
- Unsteady
- Single unit
- Multiple units
- Recycle
- By-pass
- Purge

With Chemical Reaction

- Batch
- Continous
- Semi-continuous
- Steady
- Unsteady
- Single unit
- Multiple units
- Recycle
- By-pass
- Purge

Law of Conservation of Mass

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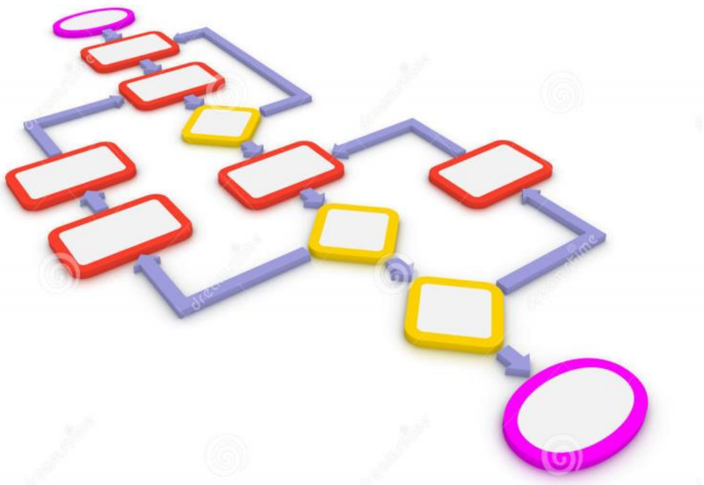
Input	+	Generation	-	Output	-	Consumption	=	Accumulation
(enters through system boundaries)		(produced within system)		(leaves through the system boundaries)		(consumed within the system)		(build up within the system

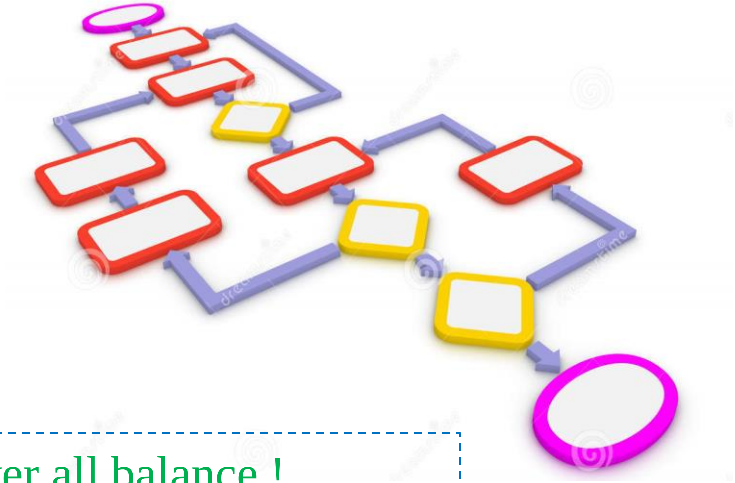
Algorithm for solving material balance problems – Without Chemical Reactions (Single units)

1. Read and understand the problem
2. Draw a sketch of the process and specify system boundary
3. Place labels (symbols, numbers and units) on the diagram for all of the known flows, materials and compositions
4. Obtain any data you need to solve the problem that are missing
5. Choose a basis
6. Determine the number of variables whose values are unknown



Algorithm for solving material balance problems – Without Chemical Reactions (Single units)

7. Determine the number of independent equations to carry out a degrees-of-freedom
8. Write down the equations to be solved in terms of known's and unknowns
9. Solve the equations and calculate the quantities
10. Check the answer



Note: It is good always to check the answer through over all balance !

2. EXAMPLE PROBLEM

Problem statement

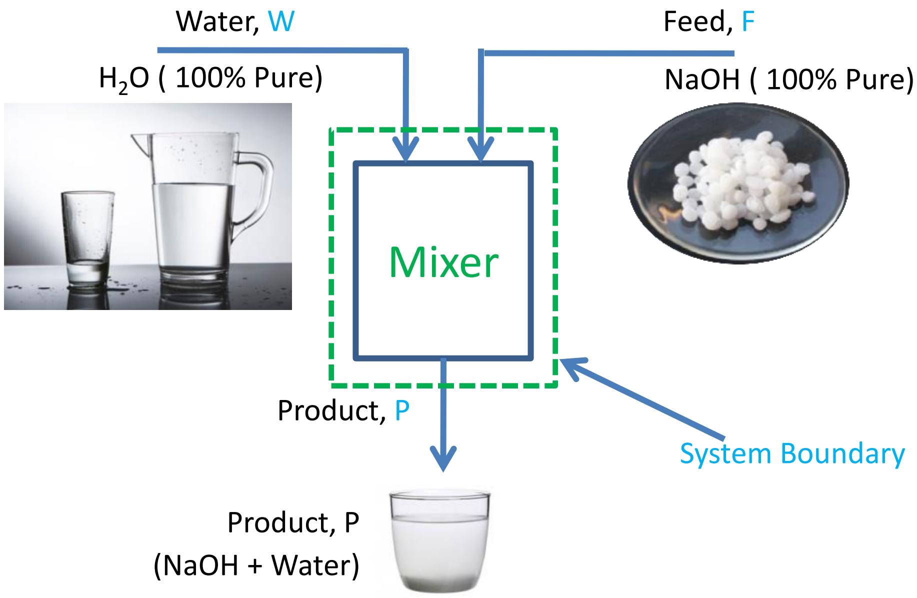
1. A continuous mixer mixes NaOH with H_2O to produce an aqueous solution of NaOH. Determine the composition and flow rate of the product if the flow rate of NaOH is 1000 kg/h and the ratio of flow rate of water to the production solution is 0.9.

2. EXAMPLE PROBLEM

Step: 01   Read and understand the problem

- Equipment
- Process
- Components in feed (Pure – Water & NaOH)
- Components in product (NaOH + Water)
- Flow rates and composition
- 1000 kg/h of NaOH in the feed
- Flow rate of water per product solution =0.9
- Flow rate of the water = 0.9 x Product solution

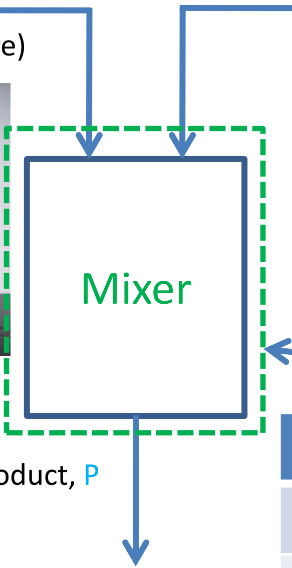
Step: 02 Draw a sketch of the process and specify system boundary



2. EXAMPLE PROBLEM

Step: 03

Draw a sketch of the process and specify system boundary



x^P_{NaOH} = mole fraction of NaOH in the product stream P

$x^P_{H_2O}$ = mole fraction of H_2O in the product stream P

P = product flow rate, kg/h

P_{H_2O} = flow rate of water in the product stream, kg/h

P_{NaOH} = flow rate of NaOH in the product stream, kg/h

Product, P
(NaOH + Water)

S. No.	Components	Kilograms (kg)	Composition
01	NaOH	P_{NaOH}	$x^P_{NaOH} = \frac{P_{NaOH}}{P}$
02	Water	P_{H2O}	$x^P_{H_2O} = \frac{P_{H_2O}}{P}$
03	Total	P	$P = 1.00$

Step: 04 Obtain any data you need to solve the problem that are missing

- Molecular Weight
- Density
- Temperature
- Pressure
- Melting Point
- Boiling Point

Note

You can look up these values in physical properties data base (For instance, Perry's Chemical Engineers Hand Book)

For our mixing problem no data is required as listed above i.e. Problem could be solvable without the above listed data.

Step: 05 Choose a basis

- Basis is the reference chosen by you for the calculations to make the problem easier to solve
- The basis may be time such as hours or a given mass of material or some other convenient quantity (mole).
- It is best to use a unit basis of 1 or 100. For instance, kilograms, hours, moles or cubic meter.
- For liquid 1 or 100 kg; similarly 1 or 100 moles is often a good choice for gases.

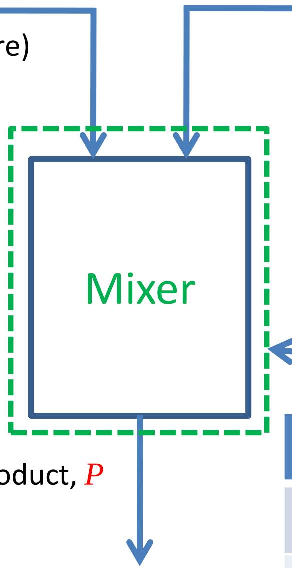
Note

Always state the basis you have chosen for your calculation by writing it prominently on your calculation sheet

Step 06: Determine the number of variables whose values are unknown

Unknown List

- W – Flow rate of water, kg/h
- P – Flow rate of Product, kg/h
- P_{H_2O} – Flow rate of water in product, kg/h
- P_{NaOH} – Flow rate of water in product, kg/h



Product, P
(NaOH + Water)

x^P_{NaOH} = mole fraction of NaOH in the product stream P

$x^P_{H_2O}$ = mole fraction of H_2O in the product stream P

P = product flow rate, kg/h

P_{H_2O} = flow rate of water in the product stream, kg/h

P_{NaOH} = flow rate of NaOH in the product stream, kg/h

S. No.	Components	Kilograms (kg)	Composition
01	NaOH	P_{NaOH}	$x^P_{NaOH} = \frac{P_{NaOH}}{P}$
02	Water	P_{H_2O}	$x^P_{H_2O} = \frac{P_{H_2O}}{P}$
03	Total	P	$P = 1.00$

Step 07: Determine the number of independent equations to carry out degrees–of–freedom.

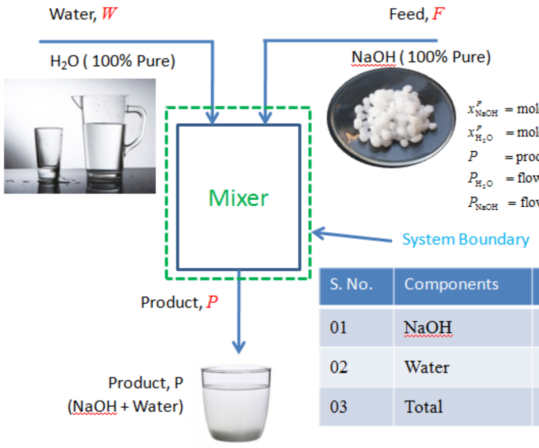
Unknown List

W – Flow rate of water, kg/h

P – Flow rate of Product, kg/h

P_{H_2O} – Flow rate of water in product, kg/h

P_{NaOH} – Flow rate of water in product, kg/h



x^P_{NaOH} = mole fraction of NaOH in the product stream P

$x^P_{H_2O}$ = mole fraction of H_2O in the product stream P

P = product flow rate, kg/h

P_{H_2O} = flow rate of water in the product stream, kg/h

P_{NaOH} = flow rate of NaOH in the product stream, kg/h

S. No.	Components	Kilograms (kg)	Composition
01	NaOH	P_{NaOH}	$x^P_{NaOH} = \frac{P_{NaOH}}{P}$
02	Water	P_{H_2O}	$x^P_{H_2O} = \frac{P_{H_2O}}{P}$
03	Total	P	$P = 1.00$

Looking at the problem you can write 3 material balances:

1. One balance equation for NaOH
2. One for H_2O
3. One Overall Balance
4. Flow rate of water/Product stream = 0.9

Step 07: Determine the number of independent equations to carry out degrees–of–freedom.

Looking at the problem you can write 3 material balances:

Note: 4 relations

1. One balance equation for NaOH

   Flow rate of NaOH in the NaOH feed stream + Flow rate of NaOH in H_2O stream = Flow rate of NaOH in product Stream

   $F_{NaOH} + W_{H_2O} = P_{NaOH}$ i.e. $F_{NaOH} + 0 = P_{NaOH}$ (Since pure NaOH is used)

2. One for H_2O

   Flow rate of H_2O in NaOH feed stream + Flow rate of H_2O in H_2O feed stream = Flow rate of H_2O in Product stream

   $F_{NaOH} + W_{H_2O} = P_{NaOH}$ i.e. $0 + W_{H_2O} = P_{NaOH}$ (Since pure H_2O is used)

3. One Overall balance

   Flow rate of NaOH in the NaOH feed stream + Flow rate of H_2O in H_2O feed stream + = Flow rate of Product stream (H_2O + NaOH)

   $F_{NaOH} + W_{H_2O} = P_{NaOH}$

4. Flow rate of Water (W)/Product stream (P) = 0.9

Step 07: Determine degrees–of–freedom

Degrees of freedom (DOF) = Number of unknowns (N_U) – Number of independent equations (N_E)

When you calculate the number of degrees of freedom you ascertain the solvability of the problem.
Three out come exists

Case	DOF	Possibility of Solution
NU = NE	0	Exactly specified (determined) solution exists
NU > NE	>0	Under specified; more independent equations required
NU < NE	<0	Over specified; more unknowns are required

Step 07: Determine degrees–of–freedom

From step 6: Number of Unknowns (N_u) = 4 (i.e. W, P, P_{H_2O} , P_{NaOH})

From step 7: Number of Independent Equations (N_E) = 4

Therefore, $DOF = N_u - N_E = 4 - 4 = 0$

Step 08: Write down the equations to be solved in terms of known's and unknowns

Note

In particular, you should attempt to write linear equations rather than nonlinear equations

Recall that the product of variables or ratios of variables or logarithm or exponent of a variable and so on in an equation causes the equation to be non linear

In many cases you can transfer the non-linear equation into linear one

For instance, in our example, the relation given $W/P = 0.9$ is a non-linear relation.

If you multiply both sides by P in above relation you get a linear equation i.e.

$$P \times W/P = P \times 0.9$$

i.e. $W = 0.9\,P$

Step 08: Write down the equations to be solved in terms of known’s and unknowns

Note

In particular, you should attempt to write linear equations rather than nonlinear equations

Overall Balance

$$F_{NaOH} + W_{H_2O} = P_{NaOH}$$

NaOH Balance

$F_{NaOH} + F_{H_2O} = P_{NaOH}$ i.e. $F_{NaOH} + 0 = P_{NaOH}$ (Since pure NaOH is used)

H_2O Balance

$F_{NaOH} + F_{H_2O} = P_{NaOH}$ i.e. $0 + F_{H_2O} = P_{NaOH}$ (Since pure H_2O is used)

Flow rate of Water(W)/Product stream(P) = 0.9 i.e. $W = 0.9P$

Step 09: Solve the equation and calculate the quantities asked

Overall Balance

$F_{NaOH} + W_{H_2O} = P_{NaOH + H_2O}$ ---- (1)

We know $W_{H_2O} = 0.9P_{NaOH + H_2O}$ substitute this relation in equation (1)

We get, $F_{NaOH} + 0.9P_{NaOH + H_2O} = P_{NaOH + H_2O}$ ---- (2)

$F_{NaOH} + 0.9P_{NaOH + H_2O} - P_{NaOH + H_2O} = 0$

$0.9P_{NaOH + H_2O} - P_{NaOH + H_2O} = F_{NaOH}$

$P\ (1-0.9) = 1000$

$P_{NaOH + H_2O} = 1000/0.1 = 10,000$ kg/h Substitute $P_{NaOH + H_2O}$ in equation (1) we get

$1000 + W_{H_2O} = 10,000$ kg/h

$W_{H_2O} = 9000$ kg/h

Step 09: Solve the equation and calculate the quantities asked

$$x_{\mathrm{NaOH}}^{P} = \frac{1000}{10000} = 0.1$$

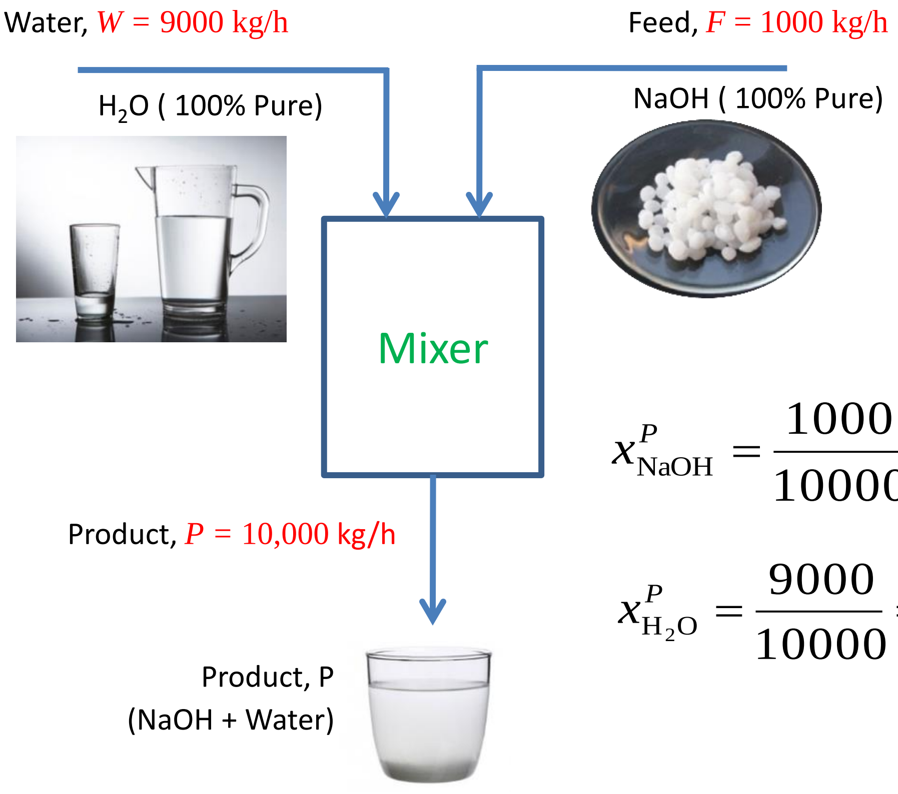
$$x_{\mathrm{H_2O}}^{P} = \frac{9000}{10000} = 0.9$$

$$F_{\mathrm{NaOH}} = 1000\ \mathrm{kg/h}$$

$$W_{\mathrm{H_2O}} = 9000\ \mathrm{kg/h}$$

$$P_{\mathrm{NaOH} + \mathrm{H_2O}} = 10000\ \mathrm{kg/h}$$

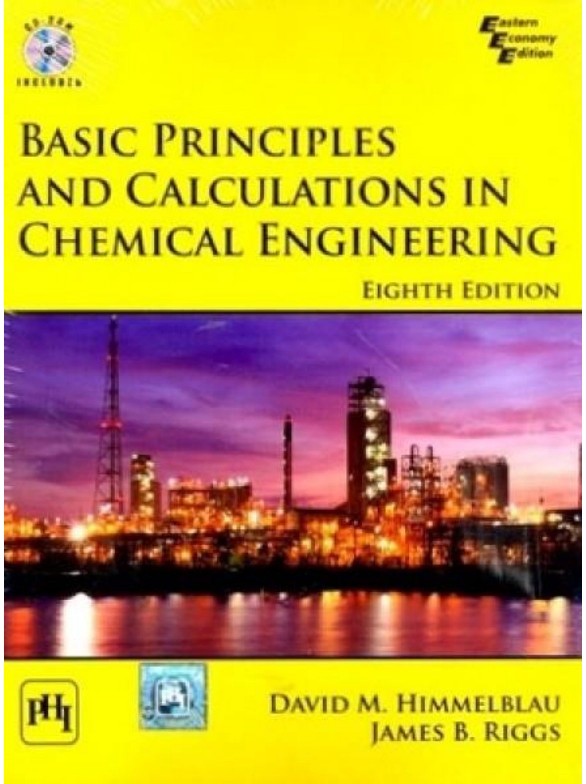
Step 10: Check the answer



$$x^{P}_{\text{NaOH}} = \frac{1000}{10000} = 0.1$$

$$x^{P}_{\text{H}_2\text{O}} = \frac{9000}{10000} = 0.9$$

References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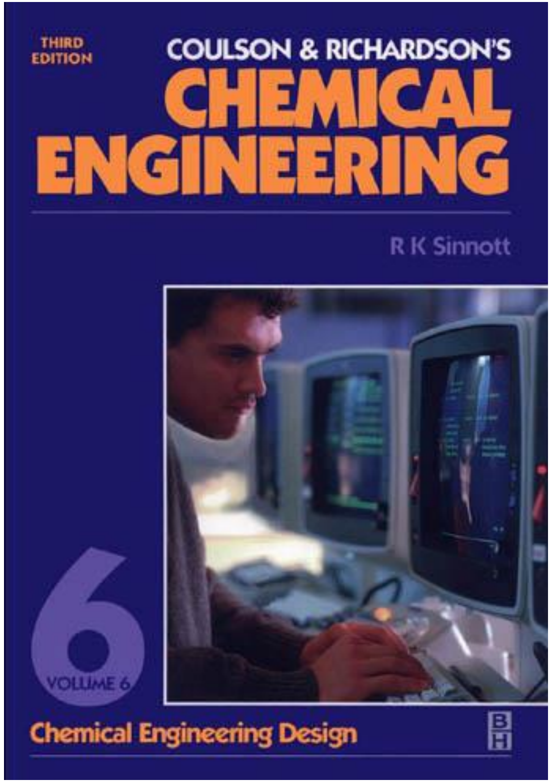
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