Introduction – Time Value of Money

Equivalence

Equations for economic studies

Amortization

Depreciation and Depletion
Introduction – Time Value of Money

1. Equivalence

2. Equations for economic studies

3. Amortization

4. Depreciation and Depletion
Process Engineering Economics

Introduction – Time Value of Money

Equivalence

Equations for economic studies

Amortization

Depreciation and Depletion
<table>
<thead>
<tr>
<th>S.No</th>
<th>Equation</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( F = P(1 + i)^n = PC_F )</td>
<td>Single payment at the end of ( n^{th} ) period</td>
</tr>
<tr>
<td>2.</td>
<td>( R = P \left( \frac{i(1+i)^n}{(1+i)^n - 1} \right) = \frac{P}{P_F} )</td>
<td>Uniform payment at the end of period (to pay fixed amount each year)</td>
</tr>
<tr>
<td>3.</td>
<td>( F = R \left( \frac{(1+i)^n - 1}{i} \right) )</td>
<td>Future worth at the end of ( n^{th} ) period</td>
</tr>
<tr>
<td>4.</td>
<td>( P = R \left( \frac{(1+i)^n - 1}{i(1+i)^n} \right) = RP_F )</td>
<td>Present Worth</td>
</tr>
<tr>
<td>S.No</td>
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</tr>
<tr>
<td>------</td>
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</tr>
<tr>
<td>4.</td>
<td>$P = R \left( \frac{(1+i)^n - 1}{i(1+i)^n} \right) = RP_F$</td>
<td>Present Worth</td>
</tr>
<tr>
<td>5.</td>
<td>$R = (P - L) \left( \frac{i(1+i)^n}{(1+i)^n - 1} \right) + L \times i$</td>
<td>Uniform payment with salvage ($L$)</td>
</tr>
<tr>
<td>6.</td>
<td>$(1+i)^n = \frac{1}{1 - \left( \frac{P}{R} \right)i}$</td>
<td>Rate of return or payment time when $L$ is zero or salvage is neglected</td>
</tr>
<tr>
<td>7.</td>
<td>$n = \frac{-\log \left( 1 - i \frac{P}{R} \right)}{\log(1+i)}$</td>
<td>Payment time when $L$ is zero or salvage is neglected</td>
</tr>
<tr>
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<td>Use</td>
</tr>
<tr>
<td>-------</td>
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<td>-----</td>
</tr>
<tr>
<td>8.</td>
<td>[ P' = \frac{R'}{i'} ]</td>
<td>Capitalized costs (or) perpetual uniform payment ( R' ) to an equivalent capital cost ( P' ) at the present time for a given interest rate.</td>
</tr>
<tr>
<td>9.</td>
<td>[ C_k = (C_{FC} - S_{FD}) f_k ]</td>
<td>Capitalized cost including cost factor.</td>
</tr>
<tr>
<td></td>
<td>[ f_k = \frac{(1 + i)^n}{(1 + i)^n - 1} ]</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>[ R'' = P \left( \frac{i'}{(1 + i')^n - 1} \right) ]</td>
<td>Sinking fund deposit, ( i' ) – is sinking fund interest rate and ( L ) is zero.</td>
</tr>
<tr>
<td>11.</td>
<td>[ P = R'' \left( \frac{(1 + i')^n - 1}{i[(1 + i')^n - 1] + i'} \right) ]</td>
<td>Hoskold’s formula - is rate of return, ( i' ) is sinking fund interest rate. Note that when ( i = i' ) equation (10) reduces to equation (4)</td>
</tr>
</tbody>
</table>
Process Engineering Economics – Equations for economic studies

\( i \) = interest rate per period
\( i' \) = sinking fund interest
\( P \) = present sum of money
\( F \) = sum at future date at ‘\( n \)’ Periods
\( R \) = end of period payment to give \( P \) in uniform series
\( L \) = salvage at some future date
\( C_F \) = compound interest factor equal to \((1 + i)^n\)
\( P_F \) = present worth factor equal to \( \frac{(1 + i)^n - 1}{i(1 + i)^n} = \frac{P}{R} \)

\( R'' \) = periodic sinking fund deposit \( R'' \)
\( R''' \) = the annual payment \( R''' \) to the owners each year which pays them when the studies of capital recovery for exploitation of mineral resources.
\( C_{FC} \) = fixed capital cost of equipment for a finite life of ‘\( n \)’ years
\( C_k \) = capitalized cost of the equipment
\( S_{FD} \) = \( \frac{S}{(1 + i)^n} \); salvage value or scrap value with compound interest
\( f_k \) = capitalized cost factor
In the above table i.e. equations used for economic studies, the compound interest factors used in all the equations from 1 to 11 are based on two series

- Single Payment series
- Uniform annual series

<table>
<thead>
<tr>
<th>Single Payment</th>
<th>Uniform annual series</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Compound-amount factor</strong></td>
<td><strong>Present-worth factor</strong></td>
</tr>
<tr>
<td><strong>Present-worth factor</strong></td>
<td><strong>Sinking-fund factor</strong></td>
</tr>
<tr>
<td><strong>Capital recovery factor</strong></td>
<td><strong>Compound-amount factor</strong></td>
</tr>
<tr>
<td><strong>Present-worth factor</strong></td>
<td><strong>Uniform annual series</strong></td>
</tr>
<tr>
<td>Given $P$ to Find $F$</td>
<td>Given $F$ to Find $P$</td>
</tr>
<tr>
<td>$(1 + i)^n$</td>
<td>$\frac{1}{(1 + i)^n}$</td>
</tr>
<tr>
<td>$\frac{i}{(1 + i')^n - 1}$</td>
<td>$\frac{i(1 + i)^n}{(1 + i)^n - 1}$</td>
</tr>
<tr>
<td>$\frac{(1 + i)^n - 1}{i}$</td>
<td>$\frac{(1 + i)^n - 1}{i(1 + i)^n}$</td>
</tr>
</tbody>
</table>
Interest formulas relating a uniform series to its present worth and future worth

We will use the relationship $F = P(1+i)^n$ in our uniform series derivation

The general relationship between $R$ and $F$ is shown in the figure given below

Where $R = \text{An end of period uniform series for n periods}$

$F = \text{Future sum or Future worth}$
Looking at the figure given below we see that if amount $R$ is invested at end of each year for 4 years, the total amount $F$ at the end of 4 years will be the sum of the compound amounts of the individual investments.

In general case for $n$ years

$$F = R \left(1+i\right)^{n-1} + \ldots + R(1+i)^3 + R(1+i)^2 + R(1+i) + R \quad \text{......... (1)}$$

Where $R =$ An end of period uniform series for $n$ periods

$F =$ Future sum or Future worth
Multiplying equation (1) by \((1+i)\), we have

\[(1+i)F = R(1+i)^n + \ldots + R(1+i)^4 + R(1+i)^3 + R(1+i)^2 + R(1+i) \ldots \ldots \ldots (2)\]

Factoring out \(R\) and subtracting equation (1) gives

\[(1+i)F = R \left[ (1+i)^n + \ldots + (1+i)^4 + (1+i)^3 + (1+i)^2 + (1+i) \right] \ldots \ldots (3)\]

\[- \quad F = R \left[ (1+i)^{n-1} + \ldots + (1+i)^3 + (1+i)^2 + (1+i) + 1 \right] \ldots \ldots (4)\]

\[iF = R \left[ (1+i)^n - 1 \right]\]

Solving above equation \(iF = R[(1+i)^n - 1]\) for \(F\) gives

\[F = R \left[ \frac{(1+i)^n - 1}{i} \right] \quad \ldots \ldots (5)\]
Thus we have an equation for $F$ when $R$ known i.e

$$F = R \left[ \frac{(1 + i)^n - 1}{i} \right]$$

---

The term inside the brackets

$$\left[ \frac{(1 + i)^n - 1}{i} \right]$$

is called uniform series compound amount factor
We know that  
\[ F = P(1 + i)^n \]
Substituting this equation for \( F \) in equation (5) we get

\[
F = R \left[ \frac{(1+i)^n - 1}{i} \right] \quad - - - - (5)
\]

\[
P(1 + i)^n = R \left[ \frac{(1+i)^n - 1}{i} \right]
\]

\[
P = R \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] \quad - - - - (6)
\]

Above equation (6) takes the form of **equation no. 4** of equations for economic studies given in the table (slide no. 6). The equation (6) can be used to calculate \( P \) if \( R \) is known. *(Nomenclature for the above equations are given in slide no. 8)*
Process Engineering Economics – Equations for economic studies

\[ F = R \left( \frac{(1+i)^n - 1}{i} \right) \quad (5) \]

Above equation (5) takes the form of equation no. 3 of equations for economic studies given in the table (slide no. 5). The equation (5) can be used to calculate \( F \) if \( R \) is known. (Nomenclature for the above equations are given in slide no. 8)

We know that

\[ P = R \left( \frac{(1+i)^n - 1}{i(1+i)^n} \right) \quad (6) \]

Rearranging the above equation (6), we have

\[ \frac{P}{R} = \left( \frac{(1+i)^n - 1}{i(1+i)^n} \right) \]

\[ R = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right] \quad (7) \]
Above equation (7) takes the form of *equation no. 2* of equations for economic studies given in the table (slide no. 5). The equation (7) can be used to calculate $R$ if $P$ is known. *(Nomenclature for the above equations are given in slide no. 8)*
Above equation (7) or *equation no. 2* of equations for economic studies given in the table (slide no. 5) can be rearranged as follows.

\[
R = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right] \quad ---- \text{(7)}
\]

\[
R[(1+i)^n - 1] = Pi(1+i)^n
\]

\[
[(1+i)^n - 1] = \frac{Pi}{R}[(1+i)^n]
\]

\[
\frac{[(1+i)^n - 1]}{(1+i)^n} = \frac{Pi}{R}
\]

\[
\frac{(1+i)^n}{(1+i)^n} - \frac{1}{(1+i)^n} = \frac{Pi}{R}
\]
Process Engineering Economics – Equations for economic studies

\[ 1 = \frac{P_i}{R} \]

\[ 1 = \frac{P_i}{R} + \frac{1}{(1 + i)^n} \]

\[ 1 - \frac{P_i}{R} = \frac{1}{(1 + i)^n} \]

\[ 1 - \frac{P_i}{R} = \frac{1}{(1 + i)^n} \]

\[ \frac{1}{P_i} = (1 + i)^n \]

\[ i.e. \ (1 + i)^n = \frac{1}{P_i} \]

\[ ----- (8) \]
(1 + i)^n = \frac{1}{Pi} \cdot \frac{1}{1 - \frac{Pi}{R}} \quad ----- (8)

Above equation (8) takes the form of *equation no. 6* of equations for economic studies given in the table (slide no. 6). The equation (8) can be used to calculate *rate of return* or *Payment time when L is zero* or *salvage/scrap value is neglected*. (*Nomenclature for the above equations are given in slide no. 8*)

Taking log on both sides of equation (8) we have

\[ n \log(1 + i) = \log(1) - \log\left(1 - \frac{Pi}{R}\right) \]

\[ n = \frac{-\log\left(1 - \frac{Pi}{R}\right)}{\log(1 + i)} \quad ----- (9) \]
Process Engineering Economics – Equations for economic studies

\[ n \log(1 + i) = \log(1) - \log \left( 1 - \frac{P_i}{R} \right) \]

\[ n = \frac{-\log \left( 1 - \frac{P_i}{R} \right)}{\log(1 + i)} \quad \text{(9)} \]

Above equation (8) takes the form of \textit{equation no. 6} of equations for economic studies given in the table (slide no. 6). The equation (8) can be used to calculate \textit{rate of return} or \textit{Payment time when L is zero} or \textit{salvage/scrap value} is neglected. \textbf{(Nomenclature for the above equations are given in slide no. 8)}
Process Engineering Economics – *References*